Answer to Question #89249 – Math – Linear Algebra

Question

An electronics company produces transistors, resistors, and computer chips. Each t ransistor requires 3 units of copper, 1 unit of zinc, and 2 units of glass. Each resisto r requires 3, 2, and 1 units of the three materials, and each computer chip requires 2 , 1, and 2 units of these materials, respectively. How many of each product can be made with 1500 units of copper, 780 units of zinc, and 1110 units of glass? Solve t his exercise by using the inverse of the coefficient matrix to solve a system of equa tions.
The company can make transistors, resistors, and computer chips.

Solution

Let the number of Transisters = x.
Let the number of Resisters = y.
Let the number of Computer Chips = z.
Given the each transistor requires 3 units of copper, 1 units of zinc, and 2 units of glass.
Given that each resistor requires 3 units of copper, 2 units of zinc, and 1 unit of glass.
Given that each computer chip requires 2 units of copper, 1 unit of zinc, and 2 units of glass.
Total number of copper units = 1500.
Total number of zinc units = 780.
Total number of glass units = 1110.
We can form the linear equations in the following way:
3(Transistors)+3(resisters)+2(computer chips)=total number of copper units

$$\Rightarrow \quad 3x + 3y + 2z = 1500 \dots\dots (1)$$

2(Transistors)+2(resisters)+1(computer chip)=total number of zinc units

$$\Rightarrow \quad x + 2y + 1z = 780 \dots\dots (2)$$

2(Transistors)+1(resisters)+2(computer chip)=total number of glass units

$$\Rightarrow \quad 2x + y + 2z = 1110 \dots\dots (3)$$

Equations (1), (2) and (3) can be written in the matrix notation as follows:

$$\begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1500 \\ 780 \\ 1110 \end{bmatrix} \dots\dots (4)$$

To find $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ we use matrix inverse method.

From (4), $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1500 \\ 780 \\ 1110 \end{bmatrix}$ ... ... (5)

Let the coefficient matrix be A = $\begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

Determinant of A = $\begin{vmatrix} 3 & 3 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{vmatrix}$

Expanding the determinant we get $|A| = 3\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$

$= 3(4-1) - 3(2-2) + 2(1-4)$ (Since, $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$)

$= 3(3) - 3(0) + 2(-3)$

$= 9 - 0 - 6$

=3

Therefore, Determinant of A is $|A| = 3 \neq 0$

A^{-1} exists.

Cofactors of an element a_{ij} of A are denoted by A_{ij}

$A_{11} = (-1)^{1+1}\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$

$= (4-1)$

$= 3$

$A_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix}$

$= -(4-4)$

$= 0$

$A_{13} = (-1)^{1+3}\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$

$= (1-4)$

$= -3$

$A_{21} = (-1)^{2+1}\begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix}$

$= -(6-2)$

$= -4$

$A_{22} = (-1)^{2+2}\begin{vmatrix} 3 & 2 \\ 2 & 2 \end{vmatrix}$

$= (6-4)$

$= 2$

$A_{23} = (-1)^{2+3}\begin{vmatrix} 3 & 3 \\ 2 & 1 \end{vmatrix}$

$= -(3-6)$

$= 3$

$A_{31} = (-1)^{3+1}\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix}$
$= (3 - 4)$
$= -1$
$A_{32} = (-1)^{3+2}\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix}$
$= -(3 - 2)$
$= -1$
$A_{33} = (-1)^{3+3}\begin{vmatrix} 3 & 3 \\ 1 & 2 \end{vmatrix}$
$= (6 - 3)$
$= 3$
The cofactors matrix is = $\begin{bmatrix} 3 & 0 & -3 \\ -4 & 2 & 3 \\ -1 & -1 & 3 \end{bmatrix}$
Adjoint A = Adj A= Transpose of cofactors matrix
= $\begin{bmatrix} 3 & 0 & -3 \\ -4 & 2 & 3 \\ -1 & -1 & 3 \end{bmatrix}^T$
$\begin{bmatrix} 3 & -4 & -1 \\ 0 & 2 & -1 \\ -3 & 3 & 3 \end{bmatrix}$

Therfore, Adj(A)=$\begin{bmatrix} 3 & -4 & -1 \\ 0 & 2 & -1 \\ -3 & 3 & 3 \end{bmatrix}$
$A^{-1} = \frac{Adj\ (A)}{\det A}$
$= \frac{1}{3}\begin{bmatrix} 3 & -4 & -1 \\ 0 & 2 & -1 \\ -3 & 3 & 3 \end{bmatrix}$
Therefore, $A^{-1} = \frac{1}{3}\begin{bmatrix} 3 & -4 & -1 \\ 0 & 2 & -1 \\ -3 & 3 & 3 \end{bmatrix}$
Substituting A^{-1}, in (5) we get $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 3 & -4 & -1 \\ 0 & 2 & -1 \\ -3 & 3 & 3 \end{bmatrix}\begin{bmatrix} 1500 \\ 780 \\ 1110 \end{bmatrix}$
$= \frac{1}{3}\begin{bmatrix} 3(1500) - 4(780) - 1110 \\ 0(1500) + 2(780) - 1110 \\ -3(1500) + 3(780) + 3(1110) \end{bmatrix}$
$= \frac{1}{3}\begin{bmatrix} 270 \\ 450 \\ 1170 \end{bmatrix}$
$= \begin{bmatrix} 90 \\ 150 \\ 390 \end{bmatrix}$

Therefore, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 90 \\ 150 \\ 390 \end{bmatrix}$, which the company produces 90 transistors, 150 resisters, and 390 computer chip.