Answer to Question #87969 – Math – Analytic Geometry

Question

Find the vertices,eccentricity,foc and asymptotes of the hyperbola x^2/8-y^2/4=1.Also trace it.Under what condition on ⋋ the line x+⋋y =2 will be tangent to this hyperbola?explain geometrically

Solution

$$\frac{x^2}{8}-\frac{y^2}{4}=1$$

Rewrite in the form of a standard hyperbola equation

$$\frac{(x-0)^2}{\left(2\sqrt{2}\right)^2}-\frac{(y-0)^2}{2^2}=1$$

$\therefore (h,\ k)=(0,\ 0),\ a=2\sqrt{2},\ b=2$

Vertices $=(0,\ 0)$

The eccentricity=e $\frac{\sqrt{a^2+b^2}}{a}$

$$e=\frac{\sqrt{\left(2\sqrt{2}\right)^2+2^2}}{2\sqrt{2}}=\sqrt{\frac{3}{2}}$$

For right-left hyperbola the asymptotes are $y=\pm\frac{b}{a}(x-h)+k$

$$y=\frac{2}{2\sqrt{2}}(x-0)+0,\qquad y=-\frac{2}{2\sqrt{2}}(x-0)+0$$

$$y=\frac{x}{\sqrt{2}},\qquad y=-\frac{x}{\sqrt{2}}$$

The Foci are $(h+c,\ k),\ (h-c,\ k)$, where $c=\sqrt{a^2+b^2}$

$$c=\sqrt{\left(2\sqrt{2}\right)^2+2^2}=\quad 2\sqrt{3}$$

Then,foci $=\left(0+2\sqrt{3},\ 0\right),\ \left(0-2\sqrt{3},\ 0\right)$

$=\left(2\sqrt{3},\ 0\right),\ \left(-2\sqrt{3},\ 0\right)$

Slope of tangent to curve $=\frac{dy}{dx}$

On differentiating,

$$\frac{x^2}{8}-\frac{y^2}{4}=1\ ....(i)$$

$$\frac{2x}{8}-\frac{2y}{4}\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{-\frac{2x}{8}}{-\frac{2y}{4}}=\frac{x}{2y}$$

From (i), $y = \pm\sqrt{4(\frac{x^2}{8}-1)}$

$y = \pm\sqrt{\frac{x^2}{2}-4}$

so, $\frac{dy}{dx} = \frac{x}{\pm 2\sqrt{\frac{x^2}{2}-4}}$

Given line : $x + \lambda y = 2$

$y = \frac{-x}{\lambda} + \frac{2}{\lambda}$

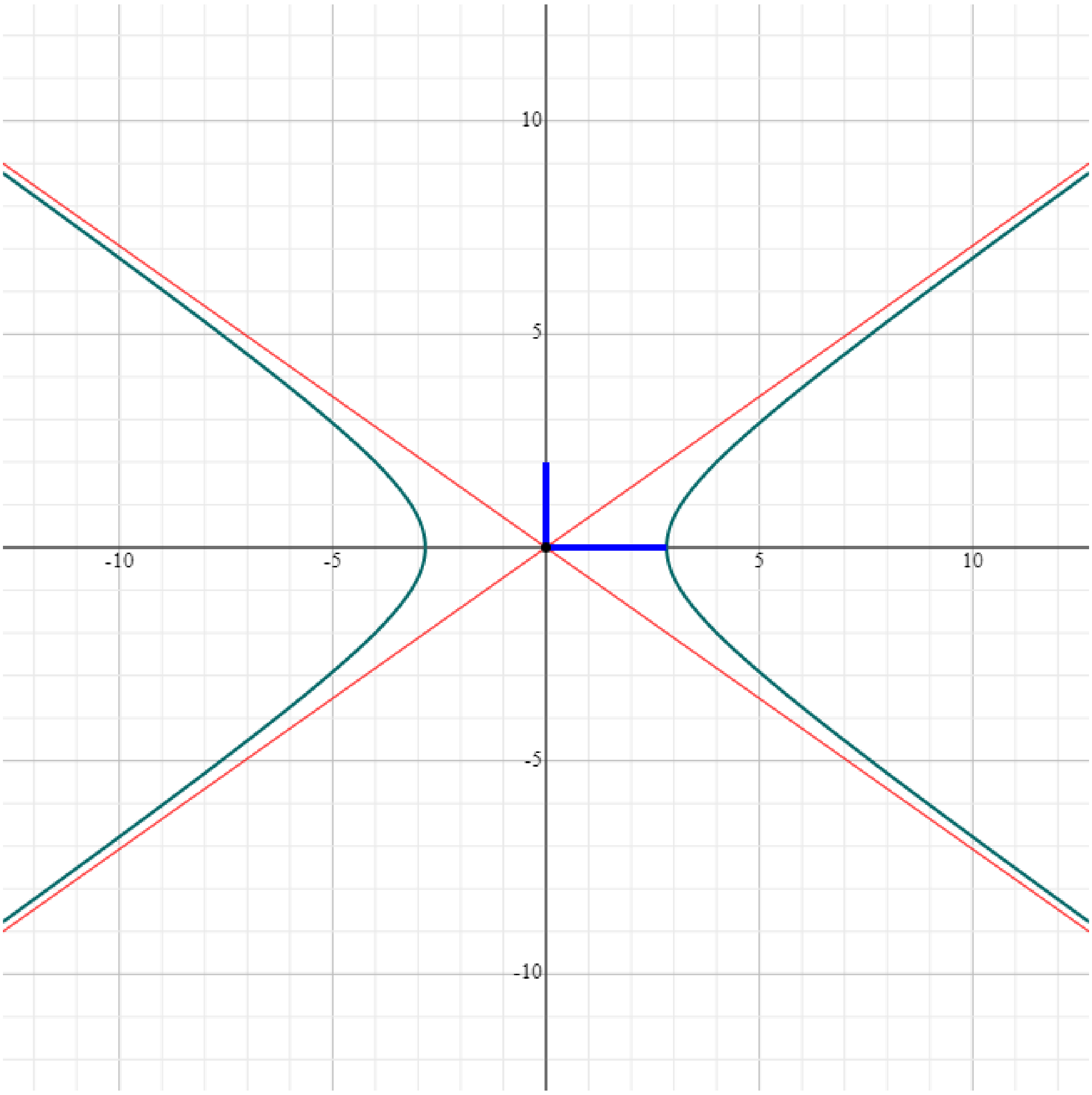
Slope $= \frac{-1}{\lambda}$

We get,

$$\frac{x}{\pm 2\sqrt{\frac{x^2}{2}-4}} = \frac{-1}{\lambda}$$

Thus, $\lambda = \mp \frac{2\sqrt{\frac{x^2}{2}-4}}{x}$

Applying these elements and tracing the hyperbola, we get following graph:



Answer provided by https://www.AssignmentExpert.com