Answer on Question #85980 – Math – Complex Analysis

Question

Describe the geometric, polar and exponential representations of

$(5i + 2)^{-1}$. (1)

Solution

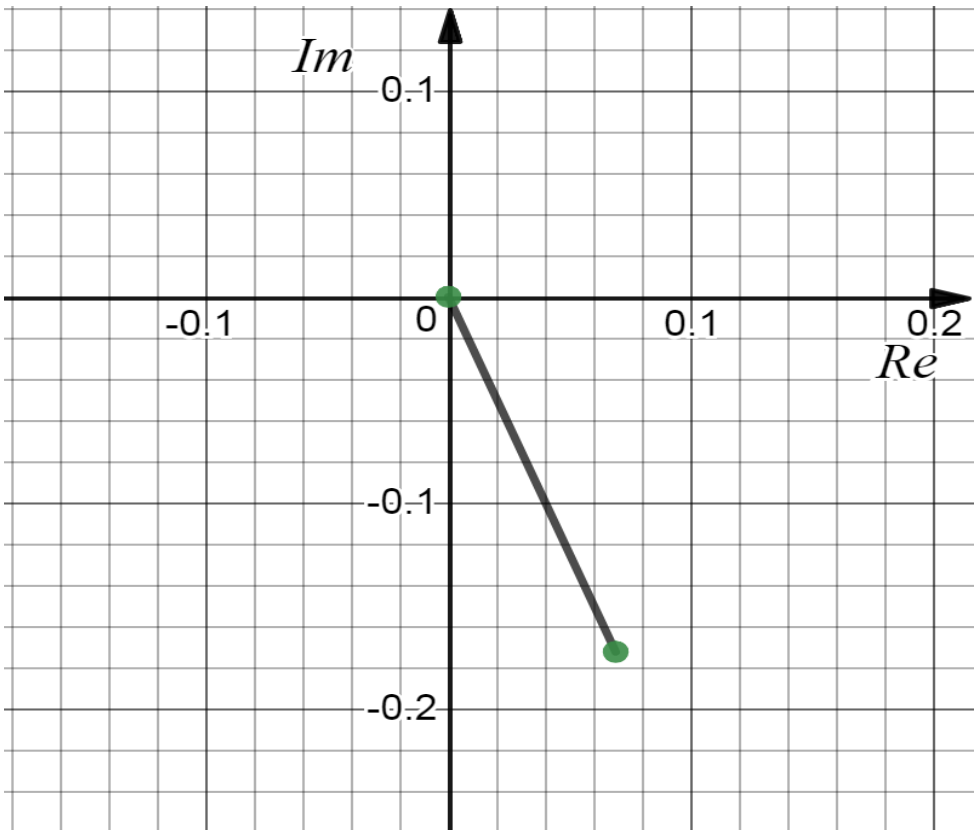
Let's write the general view of the complex number:

$z = a + ib$. (2)

Then (1) will be submitted as (2):

$z=(5i + 2)^{-1} = \frac{1}{5i+2} = \frac{5i-2}{(5i+2)(5i-2)} = \frac{5i-2}{25i^2-4} = \frac{5i-2}{-25-4} = \frac{5i-2}{-29} = \frac{2}{29} - \frac{5}{29}i$. (3)

a) the geometric representations :



b) the polar representations:

general view of the polar representations is:

$z = r\cos\varphi + ir\sin\varphi$, where $r = \sqrt{a^2 + b^2}$, $\cos\varphi = \frac{a}{r}$, $\sin\varphi = \frac{b}{r}$, then

$r = \sqrt{(\frac{2}{29})^2 + (-\frac{5}{29})^2} = \frac{\sqrt{29}}{29}$, $\cos\varphi = \frac{\frac{2}{29}}{\frac{\sqrt{29}}{29}} = \frac{2}{\sqrt{29}}$, $\sin\varphi = \frac{-\frac{5}{29}}{\frac{\sqrt{29}}{29}} = \frac{-5}{\sqrt{29}}$, $\tan\varphi = -2.5$, $\varphi \approx -68.2^{\circ}$

$z = \frac{2}{29} - \frac{5}{29}i \approx \frac{\sqrt{29}}{29}\cos(-68.2^\circ) + i\frac{\sqrt{29}}{29}\sin(-68.2^\circ) = \frac{\sqrt{29}}{29}(\cos(-68.2^\circ) + i\sin(-68.2^\circ))$.

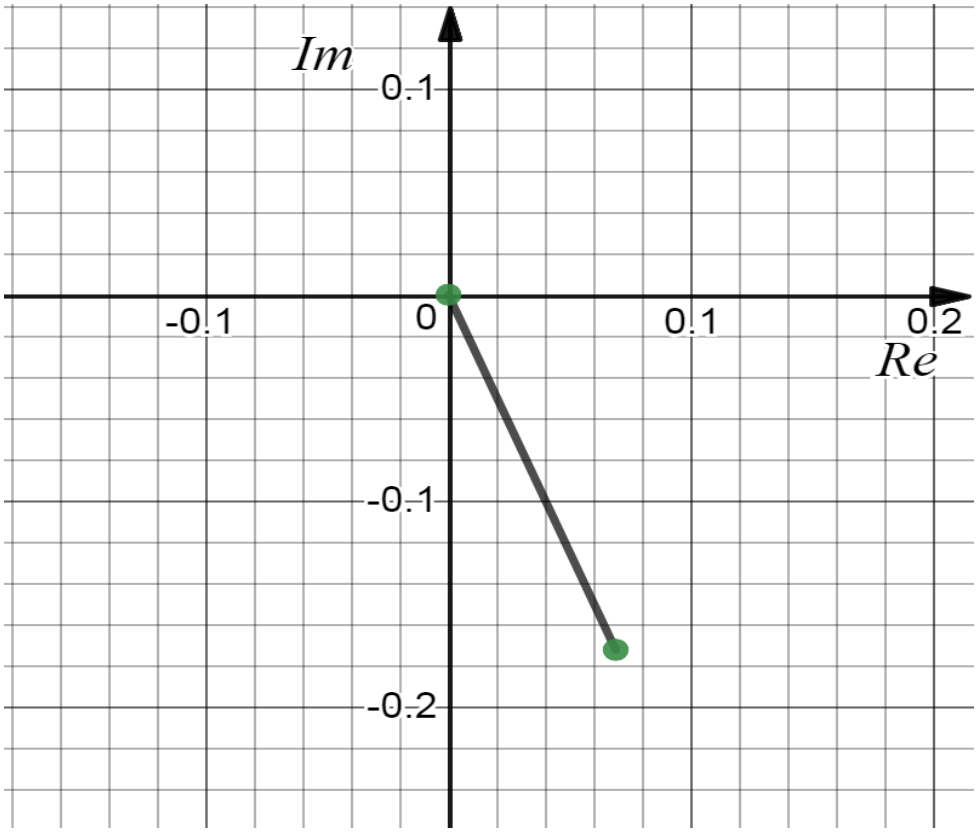
c) the exponential representations:

general view of the exponential representations is:

$z = re^{i\varphi}$, then $z \approx \frac{\sqrt{29}}{29}e^{-68.2^\circ i}$.

Answer:

a) the geometric representations of $(5i + 2)^{-1}$:



b) the polar representations of $(5i + 2)^{-1}$:

$$(5i + 2)^{-1} \approx \frac{\sqrt{29}}{29}(\cos(-68.2^\circ) - i\sin(-68.2^\circ));$$

c) the exponential representations of $(5i + 2)^{-1}$:

$$(5i + 2)^{-1} \approx \frac{\sqrt{29}}{29}e^{-68.2^\circ i}.$$