Answer on Question #85783 – Math – Quantitative Methods

Question

Perform three iterations of the inverse power method to obtain the smallest eigenvalue in magnitude of the matrix [{2,5},{3,6}].

Take appropriate initial approximation to the eigenvector.

Solution

$$A = \begin{pmatrix} 2 & 5 \\ 3 & 6 \end{pmatrix}$$

a non-zero vector ***x*** is an eigenvector of ***A*** if there exists a scalar ***λ*** such that

$$Ax = \lambda x \leftrightarrow x = A^{-1}(\lambda x) \leftrightarrow A^{-1}x = \frac{1}{\lambda}x$$

the smallest eigenvalue of ***A*** in magnitude is the largest eigenvalue of $\boldsymbol{A^{-1}}$ and we can use power method on the $\boldsymbol{A^{-1}}$ to compute its dominant eigenvalue:

$$x^{k+1} = \frac{A^{-1}x^k}{\|A^{-1}x^k\|_\infty}, \qquad x^k - k^{th} approximation\ to\ the\ dominant\ eigenvector$$

to speed up the convergence of the method, we choose the eigenvector as the initial approximation:

$$x^0 = \begin{pmatrix} -2.12 \\ 1 \end{pmatrix}, \qquad A^{-1} = \begin{pmatrix} -2 & 5/3 \\ 1 & -2/3 \end{pmatrix}$$

$$A^{-1}x^0 = \begin{pmatrix} -2 & 5/3 \\ 1 & -2/3 \end{pmatrix}\begin{pmatrix} -2.12 \\ 1 \end{pmatrix} = \begin{pmatrix} 5.907 \\ -2.787 \end{pmatrix}, \qquad x^1 = \begin{pmatrix} 1 \\ -0.472 \end{pmatrix}, \qquad |\lambda^1| = 0.169$$

$$A^{-1}x^1 = \begin{pmatrix} -2 & 5/3 \\ 1 & -2/3 \end{pmatrix}\begin{pmatrix} 1 \\ -0.472 \end{pmatrix} = \begin{pmatrix} -2.786 \\ 1.315 \end{pmatrix}, \qquad x^2 = \begin{pmatrix} -1 \\ 0.472 \end{pmatrix}, \qquad |\lambda^2| = 0.359$$

$$A^{-1}x^2 = \begin{pmatrix} -2 & 5/3 \\ 1 & -2/3 \end{pmatrix}\begin{pmatrix} -1 \\ 0.472 \end{pmatrix} = \begin{pmatrix} 2.786 \\ -1.314 \end{pmatrix}, \qquad x^3 = \begin{pmatrix} 1 \\ -0.472 \end{pmatrix}, \qquad |\lambda^3| = 0.359$$

the smallest eigenvalue of ***A*** in magnitude is 0.359.