Answer to Question #85700 – Math – Real Analysis

Function is given as $f_n(x)=\dfrac{nx}{1+n^2x^2}$

The value of function is zero when x is zero for any value of n including infinity

Which shows that the given function is constant value function?

Lets takes first derivatives of function

$$f_n^{'}(x)=\frac{(n)(1+n^2x^2)-nx(2n^2x)}{(1+n^2x^2)^2}$$

$$f_n^{'}(x)=\frac{n-n^3x^2}{(1+n^2x^2)^2}$$

In order to find out the critical point of the function we need to equate first derivative of function equal to zero

$$0=\frac{n-n^3x^2}{(1+n^2x^2)^2}$$

This gives us

$$x=\pm\frac{1}{n}$$

The value of function at this point is

$$f_n\left(\pm\frac{1}{n}\right)=\pm\frac{1}{2}$$

But at the end point

$x=\pm1$, the functions have value of

$$f_n(\pm1)=\frac{\pm1}{1+n^2}$$

In the set $[-1,1]$, maxima and minima occur either at the end points or at the critical point.

Thus, in the interval $[-1,1]$, the maximum value will be ½ for all value of n. so, Limit n tends to infinity for sup $\{\langle f_n(x) - f(x) \rangle \langle x \in [-1,1]\} = \frac{1}{2} \neq 0$

So, fn does not converges uniformly to f(x) on the $\left[-1,1\right]$.