Answer on Question #85640 – Math – Quantitative Methods

Question

Solve x^3+x^2−10x+1=0 for the root lying between 0 and 1 by the method of false position. Perform two iterations only.

Solution

The regula falsi (false position) method:

$$c_n = \frac{f(a_n)b_n - f(b_n)a_n}{f(a_n) - f(b_n)}$$

At iteration number ***n***, the number $\boldsymbol{c_n}$ is calculated as above and then, if $\boldsymbol{f(a_n)}$ and $\boldsymbol{f(c_n)}$ have the same sign, set $\boldsymbol{a_{n+1}} = \boldsymbol{c_n}$ and $\boldsymbol{b_{n+1}} = \boldsymbol{b_n}$, otherwise set $\boldsymbol{b_{n+1}} = \boldsymbol{c_n}$ and $\boldsymbol{a_{n+1}} = \boldsymbol{a_n}$ .This process is repeated until the root is approximated sufficiently well.

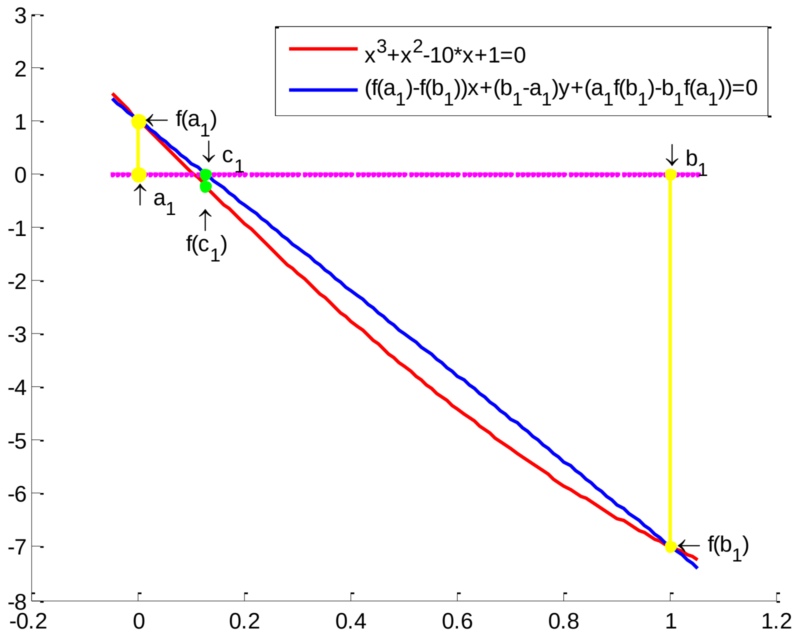
Step 1:

$$a_1 = 0, \qquad b_1 = 1$$

$$f(a_1) = f(0) = 0^3 + 0^2 - 10 \cdot 0 + 1 = 1, \qquad f(b_1) = f(1) = 1^3 + 1^2 - 10 \cdot 1 + 1 = -7$$

$$c_1 = \frac{f(a_1)b_1 - f(b_1)a_1}{f(a_1) - f(b_1)} = \frac{1 \cdot 1 - (-7) \cdot 0}{1 - (-7)} = \frac{1}{8}, \qquad f(c_1) = \left(\frac{1}{8}\right)^3 + \left(\frac{1}{8}\right)^2 - 10\left(\frac{1}{8}\right) + 1 = -\frac{119}{512}$$

$$b_2 = c_1 = \frac{1}{8}, \qquad a_2 = a_1 = 0$$

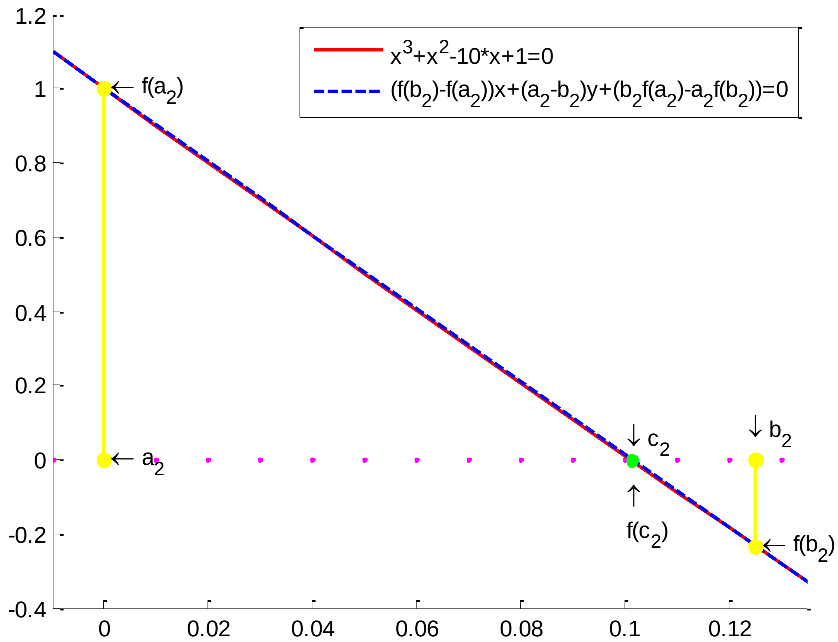


Step 2:

$$a_2 = 0, \qquad b_2 = \frac{1}{8}$$

$$f(a_2) = f(0) = 0^3 + 0^2 - 10 \cdot 0 + 1 = 1, \qquad f(b_2) = f\left(\frac{1}{8}\right) = \left(\frac{1}{8}\right)^3 + \left(\frac{1}{8}\right)^2 - 10\left(\frac{1}{8}\right) + 1 = -\frac{119}{512}$$

$$c_2 = \frac{f(a_2)b_2 - f(b_2)a_2}{f(a_2) - f(b_2)} = \frac{64}{631}, \qquad f(c_2) \cong -0.003 \;\rightarrow\; b_3 = c_2 = \frac{64}{631}, \qquad a_3 = a_2 = 0$$



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