

Question # 85540, Math / Real Analysis

Task: Are the following statements true or false? Give reasons for your answers and explain in details.

- (a) Every subsequence of the sequence $\{1/n^2\}$ is convergent.
- (b) The function $f(x) = x^2 + |x|$ is differentiable at $x = -1$.
- (c) Every infinite set is an open set.
- (d) A necessary condition for a function f to be integrable is that it is continuous.
- (e) The sum of the series $\sum_{r=1}^{3n} (1/3n + 2r)$ as $n \rightarrow \infty$ can be calculated by evaluating the integral $\int_0^3 (1/3 + 2x) dx$.

Solution:

- (a) True. A sequence converges if and only if every subsequence converges. $\{1/n^2\}$ is convergent \Rightarrow every subsequence of this sequence is convergent.
- (b) True. For $x < 0$ we have $f(x) = x^2 - x$ and $f'(x) = 2x - 1$, $f'(-1) = -3$.
- (c) False. The set $\{x: -1 \leq x \leq 1\} = [-1, 1]$ is infinite set but closed in \mathbb{R} .
- (d) True. If f is continuous then f is Riemann integrable.
- (e) False. $\sum_{r=1}^{3n} (1/3n + 2r) = 1 + 3n(3n + 1) \rightarrow \infty$ as $n \rightarrow \infty$. But $\int_0^3 (1/3 + 2x) dx = 10$.