

Answer on Question #85414 – Math – Statistics and Probability

Question

b) A typical sheet of metal has on the average 2 defects per $5 m^2$.
What is the probability that a $10 m^2$ sheet of metal will have at least 3 defects?

Solution

Assume a Poisson distribution

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}, x = 0, 1, 2, \dots$$

Let X denote the number of defects in $10 m^2$ sheet of metal. Then, since the unit area is $5 m^2$ sheet of metal, we have

$$\lambda = 2 \cdot \frac{10}{5} = 4$$

$$\begin{aligned} P(X \geq 3) &= 1 - (P(X = 0) + P(X = 1) + P(X = 2)) = \\ &= 1 - \left(\frac{4^0 e^{-4}}{0!} + \frac{4^1 e^{-4}}{1!} + \frac{4^2 e^{-4}}{2!} \right) = 1 - 13e^{-4} \approx 0.7619. \end{aligned}$$