Answer on Question #85332 – Math – Complex Analysis

Question

Obtain the harmonic conjugate v of the function u=2x(1-y)

Solution

We have

$$u_x(x,y)=\frac{\partial u(x,y)}{\partial x}=2(1-y)=\frac{\partial v(x,y)}{\partial y}$$

So $v(x,y)=2y-y^2+c(x)$, where c(x) is an arbitrary function of x

After that

$$-\frac{\partial v(x,y)}{\partial x}=-c'(x)=\frac{\partial u(x,y)}{\partial y}=-2x,$$
$$c(x)=x^2+C_1$$

So $v(x)=2y-y^2+x^2+C_1$

Answer: $v(x,y)=2y-y^2+x^2+C_1$.