

Answer to Question #85328 – Math – Complex Analysis

Locate and name of the singularities of the following functions in the finite z-plane

Question

1. $\ln(z+3i)/z^2$

Solution

1. $f(z) = \frac{\ln(z+3i)}{z^2}$

This function has two singularities one at $z=0$ of order 2 and other at $z+3i=0$ or $z=-3i$

$z=0$ is the pole of order 2.

Function of $1/z^2$ has the singularity at $z=0$, pole of order 2

Function of $\ln(z+3i)$ has a singularity point at $z=-3i$, singularity point is branch point.

Question

2. $z^2-2z/(z^2+2z+2)$

Solution

2. $f(z) = \frac{z^2 - 2z}{z^2 + 2z + 2}$

$$f(z) = \frac{z(z-2)}{(z+1)^2 + 1}$$

$$f(z) = \frac{z(z-2)}{(z+1)^2 - i^2}$$

$$f(z) = \frac{z(z-2)}{(z+1-i)(z+1+i)}$$

In order to find pole, take denominator equal to zero

Thus, $z+1-i=0$ and $z+1+i=0$

Or, $z=-1+i$ and $z=-1-i$

Function has a singularity of the pole of order 1 at $z=-1+i$ and $z=-1-i$.