Question

A batch of 5000 electric lamps has a mean life of 1000 hours and a standard deviation of 75 hours.

Assume a normal distribution.

i) How many lamps will fail before 900 hours?

ii) How many lamps will fail between 950 and 1000 hours?

iii) What proportion of lamps will fail before 925 hours?

iv) Given the same mean life, what would the standard deviation have to be to ensure that no more than 20% of lamps fail before 916 hours?

Solution

$\mu = 1000, \sigma = 75$

$Z = \dfrac{X - \mu}{\sigma} \sim N(0, 1)$

i)

$Z = \dfrac{X - \mu}{\sigma} = \dfrac{900 - 1000}{75} = -\dfrac{4}{3} \approx -1.333$

$P(X < 900) = P(Z < -1.333) = 0.0912$

$n = 0.0912 \cdot 5000 = 456 (lamps)$

ii)

$Z_1 = \dfrac{X_1 - \mu}{\sigma} = \dfrac{950 - 1000}{75} = -\dfrac{2}{3} \approx -0.667$

$Z_2 = \dfrac{X_2 - \mu}{\sigma} = \dfrac{1000 - 1000}{75} = 0$

$P(950 < X < 1000) = P(Z < 0) - P(Z < -0.667) = 0.5 - 0.2525 = 0.2475$

$n = 0.2475 \cdot 5000 \approx 1238 (lamps)$

iii)

$Z = \dfrac{X - \mu}{\sigma} = \dfrac{925 - 1000}{75} = -1$

$P(X < 900) = P(Z < -1) = 0.1587$

That is 15.87%, hence $n = 0.1587 \cdot 5000 \approx 794 (lamps)$

iv)

$\mu = 1000, \sigma = ?$

$Z = \dfrac{X - \mu}{\sigma}$

$P(X < 916) = P(Z < Z_1) = 0.2$

$Z_1 = -0.8416 = \dfrac{916 - 1000}{\sigma} => \sigma = \dfrac{916 - 1000}{-0.8416} \approx 100 (hours)$

$\sigma = 100\ hours$