Answer to Question #85241 – Math – Linear Algebra

Question

Find the inverse, if possible, for the following matrices:

(a) $\begin{pmatrix} 8 & -5 \\ 11 & 23 \end{pmatrix}$

(b) $\begin{pmatrix} 5 & -7 & 6 \\ -11 & 6 & 2 \\ 2 & 4 & -7 \end{pmatrix}$

Solution

(a) Let $A = \begin{pmatrix} 8 & -5 \\ 11 & 23 \end{pmatrix}$

Then we get,

Det A= $\begin{vmatrix} 8 & -5 \\ 11 & 23 \end{vmatrix} = (8)(23) - (-5)(11) = 184 + 55 = 239$

Therefore,

Inverse of A $= A^{-1} = \frac{Adjoint\ of\ A}{Det\ A}$

$A^{-1} = \frac{1}{239}\begin{pmatrix} 23 & -11 \\ 5 & 8 \end{pmatrix}^T$

$A^{-1} = \frac{1}{239}\begin{pmatrix} 23 & 5 \\ -11 & 8 \end{pmatrix}$

$A^{-1} = \begin{pmatrix} \frac{23}{239} & \frac{5}{239} \\ \frac{-11}{239} & \frac{8}{239} \end{pmatrix}$

(b) Let $B = \begin{pmatrix} 5 & -7 & 6 \\ -11 & 6 & 2 \\ 2 & 4 & -7 \end{pmatrix}$

Then we get,

Det B $= \begin{vmatrix} 5 & -7 & 6 \\ -11 & 6 & 2 \\ 2 & 4 & -7 \end{vmatrix}$

$Det\ B$

$= 5[6(-7) - 2(4)] + 7[(-11)(-7) - 2(2)] + 6[(-11)(4) - 2(6)]$

$= 5(-42 - 8) + 7(77 - 4) + 6(-44 - 12) = 5(-50) + 7(73) + 6(-56)$

$$= -250 + 511 - 336$$
$$= -75$$

Adjoint B

$$= \begin{pmatrix} (-1)^{1+1}\begin{vmatrix} 6 & 2 \\ 4 & -7 \end{vmatrix} & (-1)^{1+2}\begin{vmatrix} -11 & 2 \\ 2 & -7 \end{vmatrix} & (-1)^{1+3}\begin{vmatrix} -11 & 6 \\ 2 & 4 \end{vmatrix} \\ (-1)^{2+1}\begin{vmatrix} -7 & 6 \\ 4 & -7 \end{vmatrix} & (-1)^{2+2}\begin{vmatrix} 5 & 6 \\ 2 & -7 \end{vmatrix} & (-1)^{2+3}\begin{vmatrix} 5 & -7 \\ 2 & 4 \end{vmatrix} \\ (-1)^{3+1}\begin{vmatrix} -7 & 6 \\ 6 & 2 \end{vmatrix} & (-1)^{3+2}\begin{vmatrix} 5 & 6 \\ -11 & 2 \end{vmatrix} & (-1)^{3+3}\begin{vmatrix} 5 & -7 \\ -11 & 6 \end{vmatrix} \end{pmatrix}^T$$

$$= \begin{pmatrix} -50 & -73 & -56 \\ -25 & -47 & -34 \\ -50 & -76 & -47 \end{pmatrix}^T$$

$$= \begin{pmatrix} -50 & -25 & -50 \\ -73 & -47 & -76 \\ -56 & -34 & -47 \end{pmatrix}$$

We know that,

Inverse of B $= B^{-1} = \frac{Adjoint\ of\ B}{Det\ B}$

Therefore,

$$B^{-1} = \frac{1}{(-75)}\begin{pmatrix} -50 & -25 & -50 \\ -73 & -47 & -76 \\ -56 & -34 & -47 \end{pmatrix}$$

$$B^{-1} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{73}{75} & \frac{47}{75} & \frac{76}{75} \\ \frac{56}{75} & \frac{34}{75} & \frac{47}{75} \end{pmatrix}.$$

Answer:

a) $\begin{pmatrix} 8 & -5 \\ 11 & 23 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{23}{239} & \frac{5}{239} \\ \frac{-11}{239} & \frac{8}{239} \end{pmatrix};$

b) $\begin{pmatrix} 5 & -7 & 6 \\ -11 & 6 & 2 \\ 2 & 4 & -7 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{73}{75} & \frac{47}{75} & \frac{76}{75} \\ \frac{56}{75} & \frac{34}{75} & \frac{47}{75} \end{pmatrix}.$