Answer on Question #85216 – Math – Abstract Algebra

Question

If ω be the imaginary cube root of unity, show that the set $\{1, w, w^2\}$ is a cyclic group of order 3 with respect to multiplication.

Solution

1) There is a defined closed binary operation:
$w^m * w^n = w^{m+n} = w^{(m+n)\%3} \in \{1, w, w^2\}$
2) The operation is associative:
$w^m * w^n = w^{m+n} = w^{n+m} = w^n * w^m$
3) There is a unit element:
$1 * w^n = w^n * 1 = w^n$
4) All elements have inverse:
$w^n * w^{-n} = w^{n-n} = w^0 = 1$
5) There is a generator (of order 3):
$w * w * w = w^3 = 1, w * w * w * w = w^4 = w, w * w = w^2$

Therefore, it follows from 1)-4) that the set $\{1, w, w^2\}$ is a group with respect to multiplication. By 5), this group is cyclic of order 3.