Answer on Question #85215 – Math – Matrix | Tensor Analysis

Question

What are covariant and contravariant vectors? Show that acceleration is a contravariant vector.

Answer

Covariant and contravariant (from Latin co (cum) ≈ together, together, contra ≈ contrast, and vice versa vario ≈ varies) are used in mathematics (linear algebra, differential geometry, tensor analysis) and in physics concepts that characterize the way tensors (scalars, vectors, operators, bilinear forms and so on) vary in transformation bases in their respective spaces or manifolds.

Covariance and contravariance are concepts that relate to the components of the vector.

Contravariant components are "conventional" components that when changing the basis of the space are changed by conversion, an inverse transform basis.

covariant components are those that change as well as the basis.

We show that the parameters of the acceleration vector are contravariant.

Contravariant vector is a set of n ordered systems numbers mapped to each basis of this space so that the transition from the system that is associated to one basis, to the system that is associated another basis, according to the law $\alpha^{ti} = t_k^i \alpha^k$ or $\alpha^k = s_i^k \alpha^{ti}$. (In the transition from one base to the other vectors and their coordinates are converted respectively by using the inverse matrices).

On the basis of the formulas:

1) speed of movement of the point expressed by the vector method;
2) expansion of the vector on the coordinate vectors coordinate basis -

in an arbitrary curvilinear coordinate system can be expressed by the velocity vector:

$$v = \frac{dr}{dt} = e_1 \frac{dx^1}{dt} + e_2 \frac{dx^2}{dt} + e_3 \frac{dx^3}{dt} \equiv e_i \frac{dx^i}{dt}$$

By definition, this kind contravariant vector components.$v_i = \frac{dx^i}{dt}$

Find the acceleration vector in an arbitrary curvilinear coordinates:

$$w^i = \left(\frac{dv}{dt}\right)^i;$$

$$w^i = \frac{dv^i}{dt} + \Gamma_{\boldsymbol{jk}}^{\boldsymbol{i}} * v^k * \frac{dx^j}{dt};$$

$$w_i = \frac{dv_i}{dt} - \Gamma_{\boldsymbol{ji}}^{\boldsymbol{k}} * v^k * \frac{dx^j}{dt};$$

By the definition, the acceleration vector is contravariant vector as components by changing the basis of the space are changed by conversion, an inverse basis transformation.