Answer on Question #85214 – Math – Linear Algebra

Question

1. a) Show that the eigenvalues of a Hermitian matrix are real.

Solution

Let λ be eigenvalue of a hermitian matrix$A$. We have:
$Ax = \lambda x$
for some $x \neq 0$.
Since A is hermitian,
$\langle Au, v\rangle = \langle u, Av\rangle$ (1)
for all vectors u, v .
Then
$\langle Ax, x\rangle = \langle \lambda x, x\rangle = \lambda\langle x, x\rangle$,
$\langle x, Ax\rangle = \langle x, \lambda x\rangle = \bar{\lambda}\langle x, x\rangle$.
From (1) we have then:
$\lambda\langle x, x\rangle = \bar{\lambda}\langle x, x\rangle$
Since $x \neq 0, \langle x, x\rangle \neq 0$, from which
$\lambda = \bar{\lambda}$,
$\mathrm{Re}\lambda + i\mathrm{Im}\lambda = \mathrm{Re}\lambda - i\mathrm{Im}\lambda$,
$\mathrm{Im}\lambda = 0$,
which means λ is real.