Question

Find whether the following sequences converge or not

A) {2+(-1)^n}

B) (4n^3+n)/(2n^3+7n)

Solution

A) $\lim\limits_{n\to\infty}(2+(-1)^n) = \begin{cases} 2+1, & if \ n=2k \\ 2-1, & n=2k-1 \end{cases} = \begin{cases} 3, & if \ n=2k \\ 1, & n=2k-1 \end{cases}.$

$\underline{\lim\limits_{n\to\infty}}(2+(-1)^n) = 1 \neq 3 = \overline{\lim\limits_{n\to\infty}}(2+(-1)^n), hence$

hence the sequence $\{2+(-1)^n: n \geq 1\}$ does not converge.

Answer: this sequence $(\{2+(-1)^n: n \geq 1\})$ does not converge.

B) $\lim\limits_{n\to\infty}\dfrac{4n^3+n}{2n^3+7n} = \lim\limits_{n\to\infty}\dfrac{4n^2+1}{2n^2+7} = \lim\limits_{n\to\infty}\dfrac{4+\frac{1}{n^2}}{2+\frac{7}{n^2}} = \dfrac{4+0}{2+0} = 2,$ $\left(in \ other \ words, there \ exists \ \lim\limits_{n\to\infty}\dfrac{4n^3+n}{2n^3+7n}\right),$

hence the sequence $\left\{\dfrac{4n^3+n}{2n^3+7n}: n \geq 1\right\}$ converges.

Answer: this sequence $\left(\left\{\dfrac{4n^3+n}{2n^3+7n}: n \geq 1\right\}\right)$ converges.