Answer on Question #85176 – Math – Differential Equations

Question

Expand the function $f(x) = P_3{}'(x)$ in a series of the form:
$\sum_{k=0}^{\infty} A_k P_k\,(x),\, where\; P_3(x) = \frac{5x^3-3x}{2}$.
The question is based on Legendre polynomials.

Solution

If $P_3(x) = \frac{5x^3-3x}{2}$, then $f(x) = \frac{3(5x^2-1)}{2}$.

We give the function $f(x)$ in a series of Legendre polynomials, when $-1 \leq$x≤ 1 :

$f(x) = \sum_{k=0}^{\infty} A_k P_k\,(x)$, when $P_0 = 1, P_1 = x, P_2 = \frac{3x-1}{2}$, …, P_k=$\frac{1}{2^k k!}\frac{d^{k}(x^2-1)^k}{dx^k}$ – the Legendre polynomials, A_k=$\frac{2k+1`}{2}\int_{-1}^{1} f(x)\, P_k(x)dx$ – coefficients of the series .

In our case $A_k = 0, when\; k = 2m+1, m = 0$,1,2,3, …,because $f(x) = \frac{3(5x^2-1)}{2}$ is an even function.

A_0=$\frac{1`}{2}\int_{-1}^{1}(\frac{3(5x^2-1)}{2}\times 1)dx$=$\int_0^1(\frac{3(5x^2-1)}{2}$dx=($\frac{5x^3}{2}-\frac{3x}{2}$)$|_0^1$=1,

$A_2 = 5\int_0^1 \frac{3(5x^2-1)}{2}\times\frac{3x^2-1}{2}dx$ = $\frac{15}{4}\int_0^1(15x^4-5x^2-3x^2+1)dx = \frac{15}{4}(3x^5-\frac{8x^3}{3}$ +x)$|_0^1$=5,

$A_4 = 9\int_0^1 \frac{3(5x^2-1)}{2}\times\frac{35x^4-30x^2+3}{8}dx$= 9(75$\frac{x^7}{16}-111\frac{x^5}{16}+15\frac{x^3}{16}+9\frac{x}{16}$)$|_0^1$=-$\frac{27}{4}$,

…,

A_{2k}=$\frac{3(4k+1)`}{2^{2k+1}(2k)!}\int_0^1(5x^2-1)\times\frac{d^{2k}(x^2-1)^{2k}}{dx^{2k}}dx.$

Then

$\frac{3(5x^2-1)}{2}$=1+$\frac{5(3x^2-1)}{2}$ - $\frac{27(35x^4-30x^2+3)}{32}$ +…=$\sum_{2k=0}^{\infty}\frac{3(4k+1)`}{2^{4k+1}((2k)!)^2}\times\frac{d^{2k}(x^2-1)^{2k}}{dx^{2k}}\int_0^1(5x^2-1)\times\frac{d^{2k}(x^2-1)^{2k}}{dx^{2k}}dx.$

Answer: $\frac{3(5x^2-1)}{2}$=$\sum_{2k=0}^{\infty}\frac{3(4k+1)`}{2^{4k+1}((2k)!)^2}\times\frac{d^{2k}(x^2-1)^{2k}}{dx^{2k}}\int_0^1(5x^2-1)\times\frac{d^{2k}(x^2-1)^{2k}}{dx^{2k}}dx.$

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