Answer on Question #85174 – Math – Differential Equations

Question

Show that: $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cdot cos(x)$

Solution

We know that

$$J_k(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n! \cdot \Gamma(n+k+1)} \cdot \left(\frac{x}{2}\right)^{2n+k} \Longrightarrow J_{-\frac{1}{2}}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n! \cdot \Gamma\left(n - \frac{1}{2} + 1\right)} \cdot \left(\frac{x}{2}\right)^{2n-\frac{1}{2}} =$$

$$= \sqrt{\frac{2}{x}} \cdot \sum_{n=0}^{\infty} \frac{(-1)^n}{n! \cdot \Gamma\left(n + \frac{1}{2}\right)} \cdot \left(\frac{x}{2}\right)^{2n} = \left\{\Gamma\left(n + \frac{1}{2}\right) = \frac{1 \cdot 3 \cdot \ldots \cdot (2n-1)}{2^n}\sqrt{\pi} = \frac{(2n-1)!!}{2^n}\sqrt{\pi}\right\} =$$

$$= \sqrt{\frac{2}{x}} \cdot \sum_{n=0}^{\infty} \frac{(-1)^n}{n! \cdot \frac{(2n-1)!!}{2^n}\sqrt{\pi}} \cdot \frac{x^{2n}}{2^{2n}} = \sqrt{\frac{2}{\pi x}} \cdot \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n \cdot n! \cdot (2n-1)!!} \cdot x^{2n} =$$

$$= \{2^n \cdot n! \cdot (2n-1)!! = (2n)!\} = \sqrt{\frac{2}{\pi x}} \cdot \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot x^{2n} = \left\{\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot x^{2n} = \cos(x)\right\} =$$

$$= \sqrt{\frac{2}{\pi x}} \cdot cos(x).$$

Answer: $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cdot cos(x).$