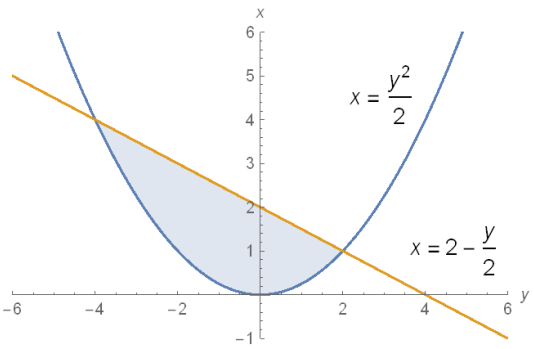
Answer on Question #85148 – Math – Calculus

Question

Find the area enclosed by the curve y^2=4x and 2x+y=4.

Solution



Instead functions $y_1(x)$ and $y_2(x)$ one can consider functions:

$$x_1(y) = \frac{1}{4}y^2, \quad x_2(y) = 2 - \frac{y}{2}$$

Now let us obtain the area enclosed by the curves $x_1(y), \quad x_2(y)$, which correspond to the shaded region on figure.

First of all we need to find the intersection points using the condition:

$$x_1(y) = x_2(y)$$

$$\frac{1}{4}y^2 - 2 + \frac{y}{2} = y^2 + 2y - 8 + 1 - 1 = (y+1)^2 - 9 = 0$$

So, we obtain $y_1 = 2, \quad y_2 = -4$

As follows from the figure: $x_2(y) \geq x_1(y)$ for $y \in [-4, 2]$.

So, the area of the shaded region is

$$S = \int_{-4}^{2} (x_2(y) - x_1(y))dy = \int_{-4}^{2} \left(2 - \frac{1}{2}y - \frac{1}{4}y^2\right)dy = \left(2 - \frac{1}{2}y - \frac{1}{4}y^2\right)\Bigg|_{-4}^{2} = \frac{7}{3} + \frac{20}{3} = 9$$

Answer: area=9.