Answer on Question #85144 – Math – Algebra

Question

An electronics company produces transistors, resistors, and computer chips. Each transistor requires 3 units of copper, 2 units of zinc, and 1 unit of glass. Each resistor requires 3, 1, and 2 units of the three materials, and each computer chip requires 2, 1, and 2 units of these materials, respectively. How many of each product can be made with 1885 units of copper, 915 units of zinc, and 1125 units of glass? Solve this exercise by using the inverse of the coefficient matrix to solve a system of equations.

Solution

Denote the number of transistors by *x*, the number of resistors by *y* and the number of computer chips by *z*. Then we obtain the system of equations:

$$\begin{cases} 3x+3y+2z=1885 \\ 2x+y+z=915 \\ x+2y+2z=1125 \end{cases}$$

$$A=\begin{pmatrix} 3 & 3 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix},\ X=\begin{pmatrix} x \\ y \\ z \end{pmatrix},\ B=\begin{pmatrix} 1885 \\ 915 \\ 1125 \end{pmatrix}$$

$$X=A^{-1}\cdot B$$

$$\Delta=\begin{vmatrix} 3 & 3 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{vmatrix}=3\cdot1\cdot2+3\cdot1\cdot1+2\cdot2\cdot2-2\cdot1\cdot1-3\cdot1\cdot2-3\cdot2\cdot2=6+3+8-2-6-12=-3$$

$\Delta\neq0$ - the system of equations has a single solution.

$$A^T=\begin{pmatrix} 3 & 2 & 1 \\ 3 & 1 & 2 \\ 2 & 1 & 2 \end{pmatrix}$$

$$\bar{a}_{11}=(-1)^2\cdot\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix}=1\cdot2-1\cdot2=0$$

$$\bar{a}_{12}=(-1)^3\cdot\begin{vmatrix} 3 & 2 \\ 2 & 2 \end{vmatrix}=-1\cdot(3\cdot2-2\cdot2)=-2$$

$$\bar{a}_{13}=(-1)^4\cdot\begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix}=3\cdot1-2\cdot1=1$$

$$\bar{a}_{21}=(-1)^3\cdot\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}=-1\cdot(2\cdot2-1\cdot1)=-3$$

$$\bar{a}_{22}=(-1)^4\cdot\begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix}=3\cdot2-2\cdot1=4$$

$$\bar{a}_{23}=(-1)^5\cdot\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix}=-1\cdot(3\cdot1-2\cdot2)=1$$

$$\bar{a}_{31}=(-1)^4\cdot\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}=2\cdot2-1\cdot1=3$$

$$\bar{a}_{32}=(-1)^5\cdot\begin{vmatrix} 3 & 1 \\ 3 & 2 \end{vmatrix}=-1\cdot(3\cdot2-3\cdot1)=-3$$

$$\bar{a}_{33} = (-1)^6 \cdot \begin{vmatrix} 3 & 2 \\ 3 & 1 \end{vmatrix} = 3 \cdot 1 - 3 \cdot 2 = -3$$

$$A^{-1} = \frac{\bar{A}}{\Delta} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -4/3 & -1/3 \\ -1 & 1 & 1 \end{pmatrix}$$

$$X = A^{-1} \cdot B = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -4/3 & -1/3 \\ -1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1885 \\ 915 \\ 1125 \end{pmatrix} = \begin{pmatrix} 0 \cdot 1885 + 2/3 \cdot 915 - 1/3 \cdot 1125 \\ 1 \cdot 1885 - 4/3 \cdot 915 - 1/3 \cdot 1125 \\ -1 \cdot 1885 + 1 \cdot 915 + 1 \cdot 1125 \end{pmatrix} = \begin{pmatrix} 235 \\ 290 \\ 155 \end{pmatrix}$$

Answer:
235 transistors, 290 resistors and 155 computer chips.