

Answer on Question #85070 – Math – Linear Algebra

Question

Apply Cramer's rule to solve the equation.

$$2x + y + z = 4$$

$$x - y + 2z = 2$$

$$3x - 2y - z = 0$$

Solution

Constructing coefficient matrix:

$$\begin{array}{ccc} 2 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & -2 & -1 \end{array}$$

Calculating determinant:

$$D = 2 \begin{vmatrix} -1 & 2 \\ -2 & -1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} + 1 \begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} = 10 + 7 + 1 = 18$$

Obtaining a matrix from, changing x column to the values on the right side of the equations given:

$$\begin{array}{ccc} 4 & 1 & 1 \\ 2 & -1 & 2 \\ 0 & -2 & -1 \end{array}$$

Calculating determinant:

$$D_x = 4 \begin{vmatrix} -1 & 2 \\ -2 & -1 \end{vmatrix} - 1 \begin{vmatrix} 2 & 2 \\ 0 & -1 \end{vmatrix} + 1 \begin{vmatrix} 2 & -1 \\ 0 & -2 \end{vmatrix} = 20 + 2 - 4 = 18$$

Obtaining a matrix from, changing y column to the values on the right side of the equations given:

$$\begin{array}{ccc} 2 & 4 & 1 \\ 1 & 2 & 2 \\ 3 & 0 & -1 \end{array}$$

Calculating determinant:

$$D_y = 2 \begin{vmatrix} 2 & 2 \\ 0 & -1 \end{vmatrix} - 4 \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} + 1 \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix} = -4 + 28 - 6 = 18$$

Obtaining a matrix from, changing z column to the values on the right side of the equations given:

$$\begin{array}{ccc} 2 & 1 & 4 \\ 1 & -1 & 2 \\ 3 & -2 & 0 \end{array}$$

Calculating determinant:

$$D_z = 2 \begin{vmatrix} -1 & 2 \\ -2 & 0 \end{vmatrix} - 1 \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix} + 4 \begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} = 8 + 6 + 4 = 18$$

Finding the values of x, y and z:

$$x = \frac{D_x}{D} = \frac{18}{18} = 1$$

$$y = \frac{Dy}{D} = \frac{18}{18} = 1$$

$$z = \frac{Dz}{D} = \frac{18}{18} = 1$$

Answer: $x = y = z = 1$.