Answer on Question #84971 – Math – Calculus

Question

Prove that the function $f(x) = 3x^4 - 4x^3 + 5$ has only one point of inflection.

Solution

We are Looking for a point of inflection:

1) the domain of this function is an interval $(-\infty; +\infty)$;
2) find the first derivative

$$f'(x) = 12x^3 - 12x^2;$$

find the second derivative

$$f''(x)=36x^2 - 24x;$$

equate the second derivative to zero:

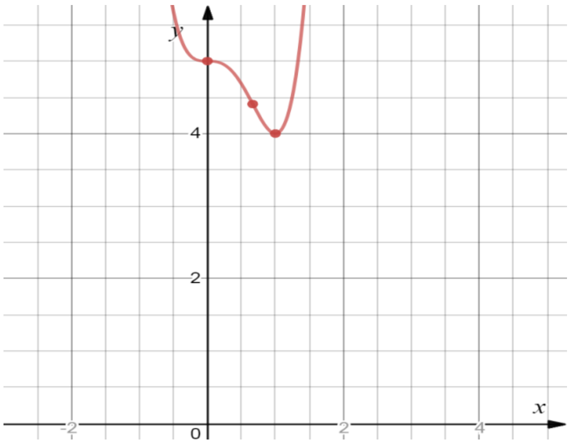
$$36x^2 - 24x = 0,$$

$$12x(3x - 2) = 0, x = 0, x = \frac{2}{3};$$

3) find the sign of the second derivative at the intervals $(-\infty; 0)$, $\left(0; \frac{2}{3}\right)$, $(\frac{2}{3}; +\infty)$:

   $f''(x) > 0$ at the interval $(-\infty; 0)$, f'' (x)<0 at the interval $\left(0; \frac{2}{3}\right)$, $f''(x) > 0$ at the interval $\left(\frac{2}{3}; +\infty\right)$.

This proves that there are two points of inflection (not one):$(0; 5) and \left(\frac{2}{3}; \frac{119}{27}\right)$. This can also be seen in the graph of the function $f(x) = 3x^4 - 4x^3 + 5$



Answer: the function $f(x) = 3x^4 - 4x^3 + 5$ has two points of inflection (not one): $(0; 5) and (\frac{2}{3}; \frac{119}{27})$.

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