ANSWER to Question #84970 – Math – Calculus

Question

Find, by the first principle, the derivative of f : R → R defined by f(x) = x^3-1 at a point x0. Hence, find the equations of the tangent and normal to its curve at the point (−2,−9).

Solution

$$f(x) = x^3 - 1$$

differentiation from first principal at $x = x_0$

$$f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h} = \lim_{h\to 0} \frac{(x_0+h)^3 - 1 - (x_0{}^3 - 1)}{h}$$

$$= \lim_{h\to 0} \frac{(x_0+h)^3 - x_0{}^3}{h}$$

$$= \lim_{h\to 0} \frac{((x_0+h) - x_0)\left((x_0+h)^2 + (x_0+h)x_0 + x_0{}^2\right)}{h}$$

$$= \lim_{h\to 0} \frac{h\left((x_0+h)^2 + (x_0+h)x_0 + x_0{}^2\right)}{h}$$

$$= \lim_{h\to 0} \left((x_0+h)^2 + (x_0+h)x_0 + x_0{}^2\right)$$

$$= \left((x_0+0)^2 + (x_0+0)x_0 + x_0{}^2\right)$$

$$= 3x_0{}^2$$

Slope of tangent at (-2, -9) = $f'(-2) = 3(-2)^2 = 12$

Hence equation of tangent at (-2, -9) will be y + 9 = 12 (x + 2) i.e. y = 12 x + 15

Slope of normal at (-2, -9) = -1/12

Hence equation of normal at (-2, -9) will be y + 9 = (-1/12) (x + 2) i.e. 12y + 108 = - x – 2

i.e. 12 y = -x – 110

Answer: $f'(x_0) = 3x_0^2;\ \ y = 12x + 15;\ \ 12y = -x - 110.$