Answer on Question #84956 – Math – Statistics and Probability

Question

We would like to estimate the difference $\mu_A - \mu_B$ in the true mean pH levels in two lakes. Samples of water specimens are collected from each lake and the pH level of each specimen is measured. Some summary statistics are shown below:

Sample	Size	Mean	Standard Deviation
Lake A	5	7.4	0.12
Lake B	4	7.1	0.28

Assume that pH levels are known to follow normal distributions for both lakes. The margin of error for a 95% confidence interval for $\mu_A - \mu_B$ is:

a. 0.324
b. 0.355
c. 0.385
d. 0.437
e. 0.477

Solution

We have that

$n_1 = 5, s_1 = 0.12, \mu_1 = 7.4$

$n_2 = 4, s_2 = 0.28, \mu_2 = 7.1$

A confidence interval for the difference in the two population means is

$$\mu_A - \mu_B = (\mu_1 - \mu_2) \pm t^*_{(n_1-1)+(n_2-1)} \cdot \sqrt{\frac{{s_p}^2}{n_1} + \frac{{s_p}^2}{n_2}}$$

$$t^*_{(n_1-1)+(n_2-1)} = t^*_{(5-1)+(4-1)} = t^*_7 = 2.365$$

$${s_p}^2 = \frac{df_1({s_1}^2) + df_1({s_1}^2)}{df_1 + df_2}$$

$${s_p}^2 = \frac{(5-1)(0.12^2) + (4-1)(0.28^2)}{5-1+4-1} = 0.0418$$

$$\sqrt{\frac{{s_p}^2}{n_1} + \frac{{s_p}^2}{n_2}} = \sqrt{\frac{0.0418}{5} + \frac{0.0418}{4}} = 0.1372$$

$$\mu_A - \mu_B = (7.4 - 7.1) \pm 2.365 \cdot 0.1372 = 0.3 \pm 0.324$$

The margin of error for a 95% confidence interval for $\mu_A - \mu_B$ is a. 0.324.

Answer: a. 0.324.