Answer on Question #84948 – Math – Differential Equations

Question

Solve the following differential equation

$$xp^2 - 2yp + ax = 0$$

Solution

$$xp^2 - 2yp + ax = 0 \qquad (*)$$

$2yp = xp^2 + ax$

$y = \dfrac{xp^2 + ax}{2p}$

$y = \dfrac{xp}{2} + \dfrac{ax}{2p}$

Differentiate both sides with respect to x

$\dfrac{dy}{dx} = \dfrac{1}{2}\left(p + x\dfrac{dp}{dx}\right) + \dfrac{a}{2}\left(\dfrac{1}{p} - \dfrac{x}{p^2}\dfrac{dp}{dx}\right)$

Then

$p = \dfrac{p}{2} + \dfrac{x}{2}\cdot\dfrac{dp}{dx} + \dfrac{a}{2p} - \dfrac{ax}{2p^2}\cdot\dfrac{dp}{dx}$

$\dfrac{dp}{dx}\left(\dfrac{x}{2} - \dfrac{ax}{2p^2}\right) = \dfrac{p}{2} - \dfrac{a}{2p}$

$x\dfrac{dp}{dx}\left(\dfrac{1}{2} - \dfrac{a}{2p^2}\right) = p\left(\dfrac{1}{2} - \dfrac{a}{2p^2}\right)$

$x\dfrac{dp}{dx} = p\,, \quad \dfrac{1}{2} - \dfrac{a}{2p^2} \neq 0$

$\dfrac{dp}{p} = \dfrac{dx}{x}$

Integrate

$\ln|p| = \ln|x| + \ln c$

$p = cx$

Substitute $p = cx$ in $(*)$

$x(cx)^2 - 2y(cx) + ax = 0$

$c^2x^3 + x(-2yc + a) = 0 => c^2x^2 - 2yc + a = 0 => 2y = cx^2 + \dfrac{a}{c}$

The general solution of the differential equation

$$2y = cx^2 + \frac{a}{c}$$

If $\dfrac{1}{2} - \dfrac{a}{2p^2} = 0,$ then $p = -\sqrt{a}$ or $p = \sqrt{a}$

Substitute $p = -\sqrt{a}$ in $(*)$

$$ax + 2\sqrt{a}y + ax = 0$$
$$y = -\sqrt{a}x$$

Substitute $p = \sqrt{a}$ in $(*)$

$$ax - 2\sqrt{a}y + ax = 0$$
$$y = \sqrt{a}x$$

The solutions of the given differential equation are

$$2y = cx^2 + \frac{a}{c}\text{ , or } y = -\sqrt{a}x\text{, or } y = \sqrt{a}x$$

Answer: $2y = cx^2 + \frac{a}{c}$, $y = -\sqrt{a}x$, $y = \sqrt{a}x$.