Question # 84927, Math / Calculus

Task: Trace the curve $y^2 = (x+1)(x-1)^2$ by showing all the properties you use to trace it.

Solution:
Of course, we have the symmetry with respect to the horizontal axis, because we have $y = \pm|x-1|\sqrt{x+1}$. Consider upper part $y = |x-1|\sqrt{x+1}$.

We have $y = 0$ if $x = \pm 1$. And if $x = 0$, then $y = 1$.

Let $x \in [-1, 1]$. Then $y = (1-x)\sqrt{x+1}$, $y' = \frac{1}{2\sqrt{x+1}} - \frac{x}{2\sqrt{x+1}} - \sqrt{x+1}$. $y' = 0 \Rightarrow x = -1/3$. So, y increases for $x \in [-1, -1/3]$ and decreases for $x \in [-1/3, 1]$. $y(-1/3) = 4/3\sqrt{2/3}$ is extremum (maximum) on $[-1, 1]$. $y' \to \infty$ if $x \to -1$, so we have vertical tangent at $x = -1$.

Let $x \in (1, \infty)$. Then $y = (x-1)\sqrt{x+1}$, $y' = -\frac{1}{2\sqrt{x+1}} + \frac{x}{2\sqrt{x+1}} + \sqrt{x+1}$. $y' > 0$, so y increases for $x \in (1, \infty)$.

Now we can trace the curve $y^2 = (x+1)(x-1)^2$.