

Answer on Question #84925 – Math – Calculus

Question

Find the length of the curve $y = \ln \left[\frac{e^x - 1}{e^x + 1} \right]$ from $x=1$ to $x=2$.

Solution

$$y = \ln \left(\frac{e^x - 1}{e^x + 1} \right), x_1 = 1, x_2 = 2.$$

$$y' = \frac{1}{\frac{e^x - 1}{e^x + 1}} \cdot \frac{(e^{2x} + e^x - e^{2x} + e^x)}{(e^x + 1)^2} = \frac{2e^x}{e^{2x} - 1}$$

$$\begin{aligned} l &= \int_{x_1}^{x_2} \sqrt{1 + (y')^2} dx = \int_1^2 \sqrt{1 + \left(\frac{2e^x}{e^{2x} - 1} \right)^2} dx = \int_1^2 \frac{e^{2x} + 1}{e^{2x} - 1} dx = \int_1^2 \left(\frac{2e^{2x}}{e^{2x} - 1} - 1 \right) dx = \\ &= (\ln(e^{2x} - 1) - x) \Big|_1^2 = \ln(e^2 + 1) - 1. \end{aligned}$$

Answer: $l = \ln(e^2 + 1) - 1$.