Answer on Question #84924 – Math – Calculus

Question

Examine for maximum and minimum values of the function
f(x)=2x^3-15x^2+36x+10

Solution

$$f(x) = 2x^3 - 15x^2 + 36x + 10$$

Find $f'(x)$ using the power rule.

$$f'(x) = 6x^2 - 30x + 36$$

The local minima and maxima will occur when the derivative equals 0:

$$6x^2 - 30x + 36 = 0$$

Divide this equation by 6

$$x^2 - 5x + 6 = 0$$

Solve this equation

$$D = (-5)^2 - 4 \cdot 1 \cdot 6 = 25 - 24 = 1$$

$$\sqrt{D} = \sqrt{1} = 1$$

$$x_1 = \frac{5+1}{2 \cdot 1} = 3, \quad x_2 = \frac{5-1}{2 \cdot 1} = 2$$

These are the two points at which maxima/minima could occur.

We identify these points using the Second Derivative Test:
Find the second derivative of the function. Then, find the sign of it at each point.
If $f''(a) < 0$ and $f'(a) = 0$, then there is a local maximum at $x = a$.
If $f''(a) > 0$ and $f'(a) = 0$, then there is a local minimum at $x = a$.

We get

$$f'(x) = 6x^2 - 30x + 36$$

$$f''(x) = 12x - 30$$

$$f''(2) = 12 \cdot 2 - 30 = -6 < 0$$

Since this is < 0 there is a local maximum at $x = 2$.

$$f''(3) = 12 \cdot 3 - 30 = 6 > 0$$

Since this is > 0 there is a local minimum at $x = 3$.

Find the values of the function at these points

$$f_{\max} = f(2) = 2 \cdot 2^3 - 15 \cdot 2^2 + 36 \cdot 2 + 10 = 16 - 60 + 72 + 10 = 38$$

$$f_{\min} = f(3) = 2 \cdot 3^3 - 15 \cdot 3^2 + 36 \cdot 3 + 10 = 54 - 135 + 108 + 10 = 37$$

Answer: $f_{\max} = f(2) = 38, \quad f_{\min} = f(3) = 37.$