Answer on Question #84873 – Math – Geometry

Question

Find an area of the composite figure with 6 sides, if I only have 4 measurements to the sides.

Solution

Consider a composite figure with 6 sides $A_1A_2A_3A_4A_5A_6$. This figure can be divided into four triangles as shown in figure 1:

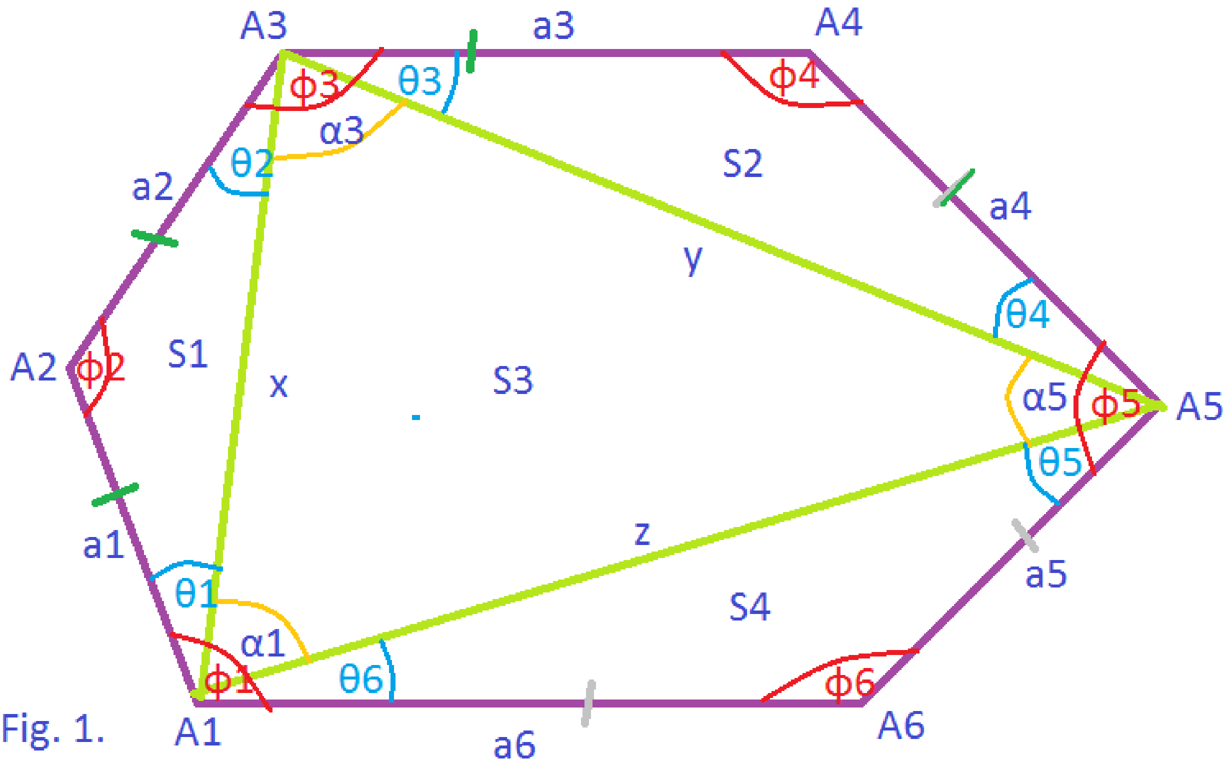

Fig. 1.

If we only have 4 sides, for example, a_1, a_2, a_3 and a_4, then an area S of $A_1A_2A_3A_4A_5A_6$ will depend on angles between the sides of the figure. If we have these angles, then we can find unknown sides and the area of each triangle:

1) From the cosine theorem in triangles $A_1A_2A_3$ and $A_3A_4A_5$:

$$x = \sqrt{(a1^2 + a2^2 - 2a1a2\cos\varphi2)};$$

$$y = \sqrt{(a3^2 + a4^2 - 2a3a4\cos\varphi4)}.$$

2) From the sine theorem in triangles $A_1A_2A_3$ and $A_3A_4A_5$:

$$x/\sin\varphi_2 = a_1/\sin\theta_2 = a_2/\sin\theta_1;$$

$$\theta_2 = \arcsin(a_1/x * \sin\varphi_2);$$

$\theta_1 = \arcsin(a_2/x * \sin\varphi_2)$;

$y/\sin\varphi_4 = a_3/\sin\theta_4 = a_4/\sin\theta_3$;

$\theta_3 = \arcsin(a_4/y * \sin\varphi_4)$;

$\theta_4 = \arcsin(a_3/y * \sin\varphi_4)$;

$\alpha_3 = \varphi_3 - (\theta_2 + \theta_3)$.

3) From the cosine theorem in the triangle $A_1A_3A_5$:

$$z = \sqrt{(x^2 + y2^2 - 2xy\cos\alpha 3)}.$$

4) From the sine theorem in the triangle $A_1A_3A_5$:

$x/\sin\alpha_5 = y/\sin\alpha_1 = z/\sin\alpha_3$;

$\alpha_1 = \arcsin(y/z * \sin\alpha_3)$;

$\alpha_5 = \arcsin(x/z * \sin\alpha_3)$;

$\theta_5 = \varphi_5 - (\theta_4 + \alpha_5)$;

$\theta_6 = \varphi_1 - (\theta_1 + \alpha_1)$.

5) From the sine theorem in the triangle $A_1A_5A_6$:

$a_6/\sin\theta_5 = a_5/\sin\theta_6 = z/\sin\varphi_6$;

$a_5 = z * \sin\theta_6/\sin\varphi_6$;

$a_6 = z * \sin\theta_5/\sin\varphi_6$.

6) Find the area of the composite figure:

$S_1 = 1/2 * a_1a_2\sin\varphi_2$;

$S_2 = 1/2 * a_3a_4\sin\varphi_4$;

$S_3 = 1/2 * xy\sin\alpha_3$;

$S_4 = 1/2 * a_5a_6\sin\varphi_6$;

$S = S_1 + S_2 + S_3 + S_4$.