# Answer on Question #84836 – Math – Combinatorics | Number Theory

## Question

f(y)=y repeated y times, for example f(3)=333,f(5)=55555.
then a=f(2001)+f(2002)+f(2003)+f(2004)+f(2005)+.......f(2012)+f(2013)+f(2014)+f(2015).
what is the remainder upon division of a by 3 ?

## Solution

If $x$ has decimal expansion $x_1x_2\ldots x_n$, then

$x(\text{mod}3) = x_1 + x_2 + \ldots + x_n(\text{mod}3)$

Indeed,

$$x = 10^{k_1}x_1 + 10^{k_2}x_2 + \ldots + x_n$$
$$= (10^{k_1} - 1)x_1 + (10^{k_2} - 1)x_2 + \ldots + (10^{k_{n-1}} - 1)x_{n-1} + (x_1 + x_2 + \ldots + x_n)$$

Since $10^k - 1$ is divisible by 3 for any $k$,

$$x(\text{mod}3) = x_1 + x_2 + \ldots + x_n(\text{mod}3)$$

This means that $f(y)(\text{mod}3) = y + y + \ldots + y = \text{y} * \text{y} = y^2(\text{mod}3)$

(here there are y summands)

Then $a(\text{mod}3) = 2001^2 + 2002^2 + 2003^2 + \ldots + 2015^2(\text{mod}3)$

We have:

2001=3*667,

from which

2001=0(mod3),

$2001^2 = 0^2(\text{mod}3), 2002^2 = 1^2(\text{mod}3), 2003^2 = 2^2(\text{mod}3), \ldots$

Then

$$a(mod3) =$$
$$= 0^2 + 1^2 + 2^2 + 0^2 + 1^2 + 2^2 + 0^2 + 1^2 + 2^2 + 0^2 + 1^2 + 2^2 + 0^2 + 1^2 + 2^2(\text{mod}3) =$$
$$= (0^2 + 1^2 + 2^2) \cdot 5(\text{mod}3) = 25(\text{mod}3) = 1(\text{mod}3)$$

**Answer:** 1.