Answer on Question #84714 – Math – Trigonometry

Question

If $\sin A = 6/10$ and $\tan B = 5/12$ where A and B are acute angles, find $\sin(A+B)$, and $\cos(2B)$ without using a calculator. All calculations must be carried out as fractions.

Solution

A and B are acute angles $=> 0 < \cos A < 1, \qquad 0 < \cos B < 1$
Pythagorean identity
$\sin^2 A + \cos^2 A = 1$
$\sin^2 B + \cos^2 B = 1$
Then

$$\begin{cases} \cos^2 A = 1 - \sin^2 A \\ 0 < \cos A < 1 \end{cases} => \cos A = \sqrt{1 - \sin^2 A} = \sqrt{1 - \left(\frac{6}{10}\right)^2} = \frac{8}{10},$$

$$\begin{cases} \dfrac{\sin^2 B}{\cos^2 B} + \dfrac{\cos^2 B}{\cos^2 B} = \dfrac{1}{\cos^2 B} \\ 0 < \cos B < 1 \end{cases} => \begin{cases} \tan^2 B + 1 = \dfrac{1}{\cos^2 B} \\ 0 < \cos B < 1 \end{cases} =>$$

$$\cos B = \sqrt{\frac{1}{\tan^2 B + 1}} = \sqrt{\frac{1}{\left(\frac{5}{12}\right)^2 + 1}} = \frac{12}{13},$$

$$\sin B = \tan B \cos B = \frac{5}{12}\left(\frac{12}{13}\right) = \frac{5}{13},$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B = \frac{6}{10}\left(\frac{12}{13}\right) + \frac{8}{10}\left(\frac{5}{13}\right) = \frac{112}{130} = \frac{56}{65},$$

$$\cos(2B) = 2\cos^2 B - 1 = 2\left(\frac{12}{13}\right)^2 - 1 = \frac{119}{169}.$$

Answer:

$\sin(A+B) = \frac{56}{65}$,

$\cos(2B) = \frac{119}{169}$.