Question

Check whether the following system of equations has a solution:

$4x + 2y + 8z + 6z = 3, 2x + 2y + 2z + 2w = 1, x + 3z + 2w = 3?$

Solution

Find (x, y, z, w) for this system
$$\begin{cases} x + 3z + 2w = 3, \\ 2x + 2y + 2z + 2w = 1, \\ 4x + 2y + 8z + 6z = 3. \end{cases}$$

Construct the following matrix and reducing it to a triangular form

$$\begin{bmatrix} 1 & 0 & 3 & 2 & 3 \\ 2 & 2 & 2 & 2 & 1 \\ 4 & 2 & 14 & 0 & 3 \end{bmatrix} \; 2\,row + 1\,row \times (-2), 3\,row + 1\,row \times (-4)$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 3 & 2 & 3 \\ 0 & 2 & -4 & -2 & -5 \\ 0 & 2 & 2 & -8 & -9 \end{bmatrix} 3\,row + 2\,row \times (-1) \rightarrow \begin{bmatrix} 1 & 0 & 3 & 2 & 3 \\ 0 & 2 & -4 & -2 & -5 \\ 0 & 0 & 6 & -6 & -4 \end{bmatrix}.$$

The system that corresponds to the last matrix has the form

$$\begin{rcases} x + 3z + 2w = 3 \\ 2y - 4z - 2w = -5 \\ 6z - 6w = -4 \end{rcases} \implies \begin{rcases} x = 3 - 3z - 2w \\ y = \frac{-5 + 4z + 2w}{2} \\ z = \frac{3w - 2}{3} \\ w = w \end{rcases} \implies \begin{rcases} x = 5 - 5w \\ y = \frac{18w - 23}{6} \\ z = \frac{3w - 2}{3} \\ w = w \end{rcases}$$

Answer: yes, $x = 5 - 5w$, $y = \frac{18w - 23}{6}$, $z = \frac{3w - 2}{3}$, $w = w$, where $w \in R$.