Answer on Question #84503 – Math – Differential Equations

Question

Task: Using the method of Laplace transform, solve the following initial value problem:

$$y'' - 3y' + 2y = 4e^{2t}$$
$$y(0) = -3$$
$$y'(0) = 5$$

Solution

$$s^2 \cdot Y(s) - s \cdot y(0) - y'(0) - 3 \cdot \big(s \cdot Y(s) - y(0)\big) + 2 \cdot Y(s) = \frac{4}{s-2}$$

$$Y(s) = \frac{(14-s)(s-2)+4}{(s-2)^2(s-1)} = \frac{-3s^2+20s-24}{(s-2)^2(s-1)} = \frac{A}{s-2} + \frac{B}{(s-2)^2} + \frac{C}{s-1} \implies$$

$$\begin{cases} A + C = -3 \\ B - 3A - 4C = 20 \\ 2A - B + 4C = -24 \end{cases} \Leftrightarrow \begin{cases} A = 4 \\ B = 4 \\ C = -7 \end{cases} \Rightarrow$$

$$Y(s) = \frac{4}{s-2} + \frac{4}{(s-2)^2} - \frac{7}{s-1}$$

Using inverse Laplace transform:

$$y(t) = e^{2t} \cdot (4 + 4t) - 7e^t$$

Answer: $y(t) = e^{2t} \cdot (4 + 4t) - 7e^t$.