

Answer to Question #84502 – Math – Differential Equations

Question

Determine the Laplace transform of the function:

$$f(t) = \begin{cases} \cos(t - 2\pi/3), & t > 2\pi/3 \\ 0, & t < 2\pi/3 \end{cases}$$

Solution

$$Y(s) = \int_{0^-}^{\infty} \cos t e^{-st} dt$$

$$\cos t = \frac{e^{it} + e^{-it}}{2}$$

$$Y(s) = \int_{0^-}^{\infty} \frac{e^{it} + e^{-it}}{2} e^{-st} dt = \frac{1}{2} \int_{0^-}^{\infty} e^{it} e^{-st} dt + \frac{1}{2} \int_{0^-}^{\infty} e^{-it} e^{-st} dt$$

$$Y(s) = \frac{1}{2(s-i)} + \frac{1}{2(s+i)} = \frac{s+i+s-i}{2(s^2+1)} = \frac{s}{s^2+1}$$

$$\cos t \leftrightarrow \frac{s}{s^2+1}$$

$$\cos\left(t - \frac{2\pi}{3}\right) \leftrightarrow e^{-\frac{2\pi}{3}s} Y(s) = e^{-\frac{2\pi}{3}s} \frac{s}{s^2+1}$$

Answer: $\cos\left(t - \frac{2\pi}{3}\right) \leftrightarrow \frac{se^{-\frac{2\pi}{3}s}}{s^2+1}$.