

Answer on Question #84501 – Math – Trigonometry

Question

Determine the Fourier transform of the function:

$$f(t) = \begin{cases} 1-t, & 0 \leq t \leq 1 \\ 1+t, & -1 \leq t \leq 0 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

Solution

Find the Fourier transform according to the formula

$$F(\omega) = \int_{-\infty}^{+\infty} f(t)e^{-i\omega t} dt, -\infty \leq \omega \leq +\infty. \quad (2)$$

Substitute in (2) the function (1):

$$\begin{aligned} F(\omega) &= \int_0^1 (1-t)e^{-i\omega t} dt + \int_{-1}^0 (1+t)e^{-i\omega t} dt + 0 = \int_0^1 e^{-i\omega t} dt - \int_0^1 te^{-i\omega t} dt + \int_{-1}^0 e^{-i\omega t} dt + \\ &\int_{-1}^0 te^{-i\omega t} dt = -\frac{e^{-i\omega}}{i\omega} + \frac{1}{i\omega} - \frac{i\omega e^{-i\omega} + e^{-i\omega} - 1}{\omega^2} - \frac{1}{i\omega} + \frac{e^{i\omega}}{i\omega} + \frac{i\omega e^{i\omega} - e^{i\omega} + 1}{\omega^2} \\ &= \frac{2i\omega \sinh(i\omega) - 2i \sinh(i\omega) - \cosh(i\omega) + 2}{\omega^2}. \end{aligned}$$

$$\text{Answer: } F(\omega) = \frac{2i\omega \sinh(i\omega) - 2i \sinh(i\omega) - \cosh(i\omega) + 2}{\omega^2}.$$