Answer on Question #84500 – Math – Complex Analysis

Question

Obtain the harmonic conjugate v of the function $u = 2x(1-y)$.

Solution

Given that $u = 2x(1-y)$

a) Prove that u is harmonic:

$$\frac{\partial u}{\partial x} = 2(1-y)$$

$$\frac{\partial u}{\partial y} = -2x$$

$$\frac{\partial^2 u}{\partial x^2} = 0$$

$$\frac{\partial^2 u}{\partial y^2} = 0$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 + 0 = 0$$

So u is harmonic.

b) Find the harmonic conjugate v of the function u. Obtain v such that u, v satisfy the Cauchy-Riemann conditions:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}; \ \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

We get $\frac{\partial v}{\partial y} = 2(1-y)$; $\frac{\partial v}{\partial x} = 2x$

From the first condition

$$v(x,y) = \int 2(1-y)dy = \int (2-2y)dy = 2y - y^2 + \varphi(x);$$

The partial derivative of x

$$\frac{\partial v}{\partial x} = \varphi'(x)$$

Substitute this in the second condition $\frac{\partial v}{\partial x} = 2x,$

we get $\varphi'(x) = 2x;\ \varphi(x) = x^2 + C, C \in R;\ v = 2y - y^2 + x^2 + C, C \in R;$

Answer: $v = 2y - y^2 + x^2 + C, C \in R;$