Answer on Question #84499 – Math – Complex Analysis

Question

Obtain the Taylor series expansion of $\cos^2 z$ about z = 0.

Solution

Apply trigonometric identity

$$\cos^2 z = \frac{1}{2}(1 + \cos 2z) \qquad (1)$$

Then we use the Taylor series expansion of $\cos z$

$$\cos z = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} + \ldots + \frac{(-1)^n z^{2n}}{(2n)!} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n}}{(2n)!}$$

Substitution $2z$ instead of z gives

$$\cos 2z = 1 - \frac{(2z)^2}{2!} + \frac{(2z)^4}{4!} - \frac{(2z)^6}{6!} + \ldots + \frac{(-1)^n (2z)^{2n}}{(2n)!} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n (2z)^{2n}}{(2n)!} \qquad (2)$$

Substitute (2) into (1), we get

$$\cos^2 z = \frac{1}{2}\left(1 + \sum_{n=0}^{\infty} \frac{(-1)^n (2z)^{2n}}{(2n)!}\right) = \frac{1}{2}\left(1 + 1 + \sum_{n=1}^{\infty} \frac{(-1)^n (2z)^{2n}}{(2n)!}\right) =$$

$$= 1 + \frac{1}{2}\sum_{n=1}^{\infty} (-1)^n 2^{2n} \frac{z^{2n}}{(2n)!} = 1 + \sum_{n=1}^{\infty} (-1)^n 2^{2n-1} \frac{z^{2n}}{(2n)!}$$

Answer: $\cos^2 z = 1 + \sum_{n=1}^{\infty} (-1)^n 2^{2n-1} \frac{z^{2n}}{(2n)!}$.