Locate and name the singularities of the following functions in the finite z-plane:

1. $f_1(z) = \frac{\ln(z+3i)}{z^2}$
2. $f_2(z) = \frac{z^2-2z}{z^2+2z+2}$

Answer:

1. the function $f_1(z)$ has 2 singularities at points: $-3i$ and 0.
2. the function $f_2(z)$ has 2 singularities at points: $-1+i$ and $-1-i$.

1 $f_1(z)$

A function $\ln(z)$ has a singularity at $z = 0$. This type of singularity points is also called a branch point. It means that as we travel from the point to itself looping around the branch point, we will eventually change the function value.

Now we know that $\ln(z+3i)$ has a singularity point at $z = -3i$. In the vicinity of this point function $\frac{1}{z^2}$ is analytical. So the multiplication of those two will still has a singularity.

A function $\frac{1}{z^2}$ has a singularity at $z = 0$, namely a pole of order 2. In the vicinity of this point function $\ln(z+3i)$ is analytical. As in the previous case, we conclude that $f_1(z)$ has one more singularity.

As the function $f_1(z)$ is analytic everywhere except those two points, we now conclude that no more singularities exist.

2 $f_2(z)$

Let's start with transforming the function:

$$\frac{z^2-2z}{z^2+2z+2} = \frac{z(z-2)}{(z+1)^2+1}$$

$$= \frac{z(z-2)}{(z+1)^2-i^2} = \frac{z(z-2)}{(z+1-i)(z+1+i)}$$

$$= z\frac{1+3i}{2(z+1-i)} + z\frac{1-3i}{2(z+1+i)}$$

Now we see that at points $-1+i$ and $-1-i$ function $f_2(z)$ has poles of order 1.