Answer on Question #84395 – Math – Differential Equations

Question

Form the differential equation having $y = (\sin^{-1} x)^2 + A\cos^{-1} x + B$, where A and B are arbitrary constants, as its general solution.

Solution

We have

$$y = (\sin^{-1} x)^2 + A\cos^{-1} x + B$$

Differentiating the given function w.r.t. x successively, we get

$$\frac{dy}{dx} = 2\sin^{-1} x\left(\frac{1}{\sqrt{1-x^2}}\right) + A\left(-\frac{1}{\sqrt{1-x^2}}\right)$$

Hence

$$\sqrt{1-x^2}\frac{dy}{dx} = 2\sin^{-1} x - A$$

On again differentiating w.r.t. x , we get

$$\sqrt{1-x^2}\frac{d^2y}{dx^2} + \left(-\frac{2x}{2\sqrt{1-x^2}}\right)\frac{dy}{dx} = 2\left(\frac{1}{\sqrt{1-x^2}}\right)$$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$$

This is the required differential equation.

Answer: $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$