Answer to Question #84284, Math / Calculus

Question: Find the asymptote of curve $x^3 - 4xy^2 - 3x^2 + 12xy - 12y^2 + 8x + 2y + 4 = 0$.

Solution:

To find the rectangular asymptotes, equate the coefficient of $x^3 = 0$ and coefficient of $y^2 = 0$.

The first implies 1 = 0 which is absurd. Therefore there is no asymptote parallel to *x*-axis.

The second implies $-4x - 12 = 0 \Rightarrow x = -3$, hence $x = -3$ is an asymptote parallel to *y*-axis.

Let the oblique asymptote be $y = mx + c$.

Now to find the oblique asymptote, put $x = 1$ and $y = m$.

We get $\phi_3(m) = 1 - 4m^2 = 0 \Rightarrow m = \pm\frac{1}{2}$.

Then $c\phi_3'(m) + \phi_2(m) = 0 \Rightarrow c(-8m) + \left(-3 + 12m - 12m^2\right) = 0$.

When $m = \frac{1}{2}$, $c(-4) + (-3 + 6 - 3) = 0 \Rightarrow c = 0$.

When $m = -\frac{1}{2}$, $c(4) + (-3 - 6 - 3) = 0 \Rightarrow c = 3$.

Hence the oblique asymptotes are $y = \frac{1}{2}x \Rightarrow x - 2y = 0$ and $y = -\frac{1}{2}x + 3 \Rightarrow x + 2y = 6$.

$x - 2y = 0$ and $x + 2y = 6$.