Answer on Question #84254 – Math – Calculus

Question

Q1. A particle is positioned at the origin of a set of axes. Two forces act on it. The first has magnitude 5 N and acts in the direction of the negative x-axis. The second has magnitude 12 N and acts in the direction of the positive y-axis. Calculate the magnitude and direction of the resultant force.

Solution

$$|\vec{F}_1| = 5\ N,\ \ |\vec{F}_2| = 12\ N$$

1. The magnitude of resultant force:

$$|\vec{F}_{res}| = \sqrt{|\vec{F}_1|^2 + |\vec{F}_2|^2} = \sqrt{5^2 + 12^2} = \sqrt{169} = 13\text{ N}$$

2. Angle between $\vec{F}_{res}$ and negative direction of x-axis:

$$\alpha = \tan^{-1}\frac{|\vec{F}_2|}{|\vec{F}_1|} = \tan^{-1}\frac{12\ N}{5\ N} = \tan^{-1} 2.4 \approx 67.38^\circ$$

Answer: magnitude: 13 N, direction: 67.38°.

Question

Q2. If a = 5i - 2j and b = -2i + 2j, find unit vectors of vectors a, b and a - b.

Solution

$$\vec{a} = 5\vec{i} - 2\vec{j}; \qquad \vec{b} = -2\vec{i} + 2\vec{j};$$

1. Unit vector of $\vec{a}$:

$$\vec{e} = \frac{\vec{a}}{|\vec{a}|} = \frac{5\vec{i}-2\vec{j}}{\sqrt{a_x^2 + a_y^2}} = \frac{5\vec{i}-2\vec{j}}{\sqrt{5^2+(-2)^2}} = \frac{5\vec{i}-2\vec{j}}{\sqrt{25+4}} = \frac{5\vec{i}-2\vec{j}}{\sqrt{29}} = \frac{5}{\sqrt{29}}\vec{i} - \frac{2}{\sqrt{29}}\vec{j}$$

2. Unit vector of $\vec{b}$:

$$\vec{e} = \frac{\vec{b}}{|\vec{b}|} = \frac{-2\vec{i}+2\vec{j}}{\sqrt{b_x^2 + b_y^2}} = \frac{-2\vec{i}+2\vec{j}}{\sqrt{(-2)^2+2^2}} = \frac{-2\vec{i}+2\vec{j}}{\sqrt{4+4}} = \frac{-2\vec{i}+2\vec{j}}{\sqrt{2*4}} = -\frac{2}{2\sqrt{2}}\vec{i} + \frac{2}{2\sqrt{2}}\vec{j} = -\frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$$

3. Unit vector of $\vec{a} - \vec{b}$:

$$\vec{a} - \vec{b} = (a_x - b_x)\vec{i} + (a_y - b_y)\vec{j} = (5 - (-2))\vec{i} + (-2-2)\vec{j} = 7\vec{i} - 4\vec{j}$$

$$\vec{e} = \frac{\vec{a}-\vec{b}}{|\vec{a}-\vec{b}|} = \frac{7\vec{i}-4\vec{j}}{\sqrt{(7)^2+(-4)^2}} = \frac{7\vec{i}-4\vec{j}}{\sqrt{49+16}} = \frac{7\vec{i}-4\vec{j}}{\sqrt{65}} = \frac{7}{\sqrt{65}}\vec{i} - \frac{4}{\sqrt{65}}\vec{j}$$

Answer: unit vectors: $\frac{5}{\sqrt{29}}\vec{\imath} - \frac{2}{\sqrt{29}}\vec{\jmath}$, $-\frac{1}{\sqrt{2}}\vec{\imath} + \frac{1}{\sqrt{2}}\vec{\jmath}$, $\frac{7}{\sqrt{65}}\vec{\imath} - \frac{4}{\sqrt{65}}\vec{\jmath}$

Question

Q3. If u = 3i + 2j and v = -5i + 4j.
(a) Express i and j in terms of u and v.
(b) Suppose i + 8j = au + bv, find the values of a and b

Solution

$\vec{u} = 3\vec{\imath} + 2\vec{\jmath}, \quad \vec{v} = -5\vec{\imath} + 4\vec{\jmath}$

$(\boldsymbol{a})$

$$-2\vec{u} = -6\vec{\imath} - 4\vec{\jmath}, \quad \vec{v} = -5\vec{\imath} + 4\vec{\jmath} \longrightarrow -2\vec{u} + \vec{v} = -11\vec{\imath} \longrightarrow \vec{\imath} = \frac{2}{11}\vec{u} - \frac{1}{11}\vec{v}$$

$$\vec{u} = 3\vec{\imath} + 2\vec{\jmath} = \frac{6}{11}\vec{u} - \frac{3}{11}\vec{v} + 2\vec{\jmath} \longrightarrow 2\vec{\jmath} = \frac{11}{11}\vec{u} - \frac{6}{11}\vec{u} + \frac{3}{11}\vec{v} = \frac{5}{11}\vec{u} + \frac{3}{11}\vec{v} \longrightarrow \vec{\jmath} = \frac{5}{22}\vec{u} + \frac{3}{22}\vec{v}$$

$(\boldsymbol{b})$

$$\vec{\imath} + 8\vec{\jmath} = \mathrm{a}\vec{u} + \mathrm{b}\vec{v}$$

$$\vec{\imath} + 8\vec{\jmath} = \frac{2}{11}\vec{u} - \frac{1}{11}\vec{v} + \frac{20}{11}\vec{u} + \frac{12}{11}\vec{v} = 2\vec{u} + \vec{v} = \mathrm{a}\vec{u} + \mathrm{b}\,\vec{v} \longrightarrow \mathrm{a} = 2, \mathrm{b} = 1$$

Answer:

(a) $\vec{\imath} = \frac{2}{11}\vec{u} - \frac{1}{11}\vec{v}$, $\vec{\jmath} = \frac{5}{22}\vec{u} + \frac{3}{22}\vec{v}$
(b) a = 2, b = 1

Question

Q4. If a = 3i - 2j and b = 2i - 3j, find

(i) |a|; **(ii)** b; **(iii)** |b - a|; **(iv)** |a - b|.

Solution

$\vec{a} = 3\vec{\imath} - 2\vec{\jmath}, \quad \vec{b} = 2\vec{\imath} - 3\vec{\jmath}$

(i) $|\vec{a}| = \sqrt{a_x^2 + a_y^2} = \sqrt{3^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$

(ii) $|\vec{b}| = \sqrt{b_x^2 + b_y^2} = \sqrt{2^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}$

(iii) $\vec{b} - \vec{a} = (b_x - a_x)\vec{\imath} + (b_y - a_y)\vec{\jmath} = -\vec{\imath} - \vec{\jmath}$

$$|\vec{b} - \vec{a}| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$$

(iv) $\left|\vec{a} - \vec{b}\right| = \left|\vec{b} - \vec{a}\right| = \sqrt{2}$

Answer: $|\vec{a}| = \left|\vec{b}\right| = \sqrt{13}$, $\left|\vec{a} - \vec{b}\right| = \left|\vec{b} - \vec{a}\right| = \sqrt{2}$.