Answer on Question #84157 – Math – Geometry

Question

ABCD is a square. E,F are points on BC,CD such that ∠EAF=45°, ∠EAB=15°. BD intersects AE at P. What is the value of ∠ PFC in degrees?

Solution

∠ABP =∠ DBC = 45° because BD is the bisector of ∠ ABC (∠ABC = 90° because ABCD is a square).

∠APB = 180° – ∠BAP – ∠ABP = 180° – 15° – 45° = 120° (by theorem about the sum of angles of a triangle).

ΔABP = ΔBPC by the SAS theorem (PB is common, AB=BC as the sides of the square and the angle between them), then ∠BPC = ∠APB = 120°.

∠BPC and ∠DPC are adjacent angles, then
∠CPD = 180° – ∠BPC = 180° – 120° = 60°.

Δ DPC = ΔDPA by SAS theorem (AD=DC, ∠PDC = ∠PDA, PD is common), then
∠PCD = ∠PAD = ∠DAF + ∠FAP = 30°+45°= 75°.
The triangles ∆AOP and ∆DOF are similar (∠ AOP = ∠DOF as vertical, ∠OAP = ∠ODF = 45°), then $\frac{OF}{OP} = \frac{OD}{OA}$.

The ΔADO and the ΔPFO are similar triangles (∠AOD and ∠ POF are equal as vertical, $\frac{OF}{OP} = \frac{OD}{OA}$ because the triangles ∆AOP and ∆DOF are similar) then ∠OFP = ∠ODA = 45°, in other words, ∠AFP = ∠ADB = 45°.

∠AOD = 180° – ∠DAO – ∠ADO = 180°-30°-45°=105° (by theorem about the sum of angles of a triangle)
∠FOD = 180° – ∠AOD = 180° – 105° = 75°
∠AFD = 180° – ∠DOF – ∠ODF = 180° – 75° – 45°=60°
∠PFD = $\angle PFO + \angle OFD$ = 45° + 60° = 105°.
∠PFC = 180° – ∠PFD = 180° – 105° = 75°.

Answer: ∠PFC = 75°.