

Answer on Question #84157 – Math – Geometry

Question

ABCD is a square. E,F are points on BC,CD such that $\angle EAF=45^\circ$, $\angle EAB=15^\circ$. BD intersects AE at P. What is the value of $\angle PFC$ in degrees?

Solution

$\angle ABP = \angle DBC = 45^\circ$ because BD is the bisector of $\angle ABC$ ($\angle ABC = 90^\circ$ because ABCD is a square).

$\angle APB = 180^\circ - \angle BAP - \angle ABP = 180^\circ - 15^\circ - 45^\circ = 120^\circ$ (by theorem about the sum of angles of a triangle).

$\triangle ABP = \triangle BPC$ by the SAS theorem (PB is common, AB=BC as the sides of the square and the angle between them), then $\angle BPC = \angle APB = 120^\circ$.

$\angle BPC$ and $\angle DPC$ are adjacent angles, then
 $\angle CPD = 180^\circ - \angle BPC = 180^\circ - 120^\circ = 60^\circ$.

$\triangle DPC = \triangle DPA$ by SAS theorem (AD=DC, $\angle PDC = \angle PDA$, PD is common), then
 $\angle PCD = \angle PAD = \angle DAF + \angle FAP = 30^\circ + 45^\circ = 75^\circ$.

The triangles $\triangle AOP$ and $\triangle DOF$ are similar ($\angle AOP = \angle DOF$ as vertical, $\angle OAP = \angle ODF = 45^\circ$), then
 $\frac{OF}{OP} = \frac{OD}{OA}$.

The $\triangle ADO$ and the $\triangle PFO$ are similar triangles ($\angle AOD$ and $\angle POF$ are equal as vertical, $\frac{OF}{OP} = \frac{OD}{OA}$ because the triangles $\triangle AOP$ and $\triangle DOF$ are similar) then $\angle OFP = \angle ODA = 45^\circ$, in other words,
 $\angle AFP = \angle ADB = 45^\circ$.

$\angle AOD = 180^\circ - \angle DAO - \angle ADO = 180^\circ - 30^\circ - 45^\circ = 105^\circ$ (by theorem about the sum of angles of a triangle)

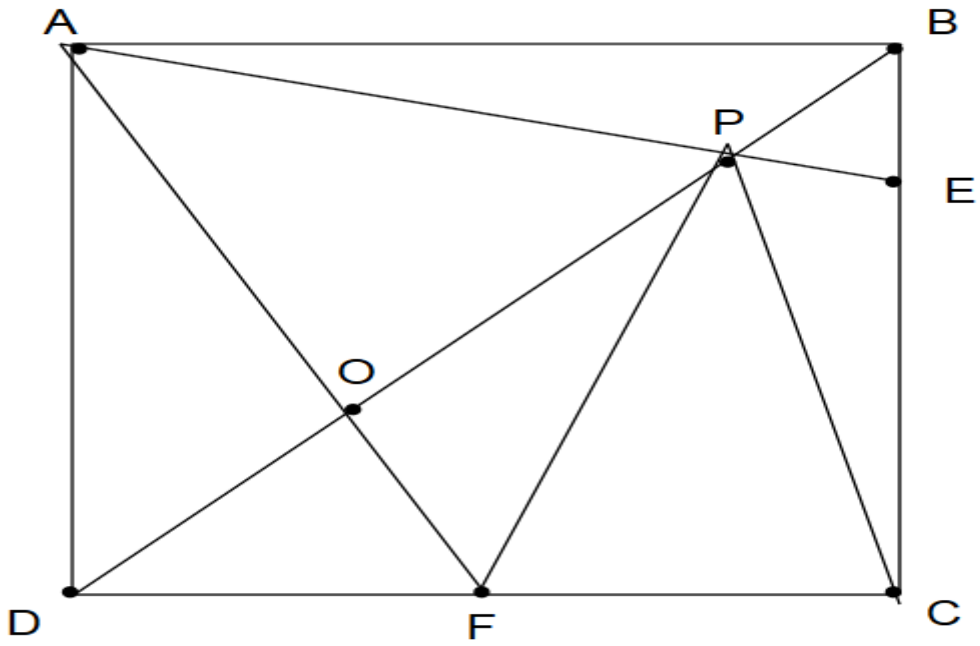
$\angle FOD = 180^\circ - \angle AOD = 180^\circ - 105^\circ = 75^\circ$

$\angle AFD = 180^\circ - \angle DOF - \angle ODF = 180^\circ - 75^\circ - 45^\circ = 60^\circ$

$\angle PFD = \angle PFO + \angle OFD = 45^\circ + 60^\circ = 105^\circ$.

$\angle PFC = 180^\circ - \angle PFD = 180^\circ - 105^\circ = 75^\circ$.

Answer: $\angle PFC = 75^\circ$.



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