

Answer on Question #84078 – Math – Real Analysis

Question

- a) Every infinite set is an open set.
- b) A necessary condition for a function f to be integrable is that it is continuous.

true or false?

Solution

- a) False. A counterexample can be the set of integer numbers $\{\dots, -2, -1, 0, 1, 2, \dots\}$ on the real axis, which is infinite as a set of points but not open (it is closed, in fact), or intervals of type $[a, b)$, $(a, b]$, $[a, b]$ with $a < b$, which are infinite as sets of points, but not open.
- b) False. The function $f(x)$ such that $f(x) = 0$ for $x \leq 0$, and $f(x) = 1$ for $x > 0$ is integrable on every finite interval, but it is not continuous at $x = 0$.

Answer: a) false; b) false.