

Answer on Question #84076 – Math – Real Analysis

Question

The function $f(x) = x^2 + x$ is differentiable at $x = -1$.

Solution

A function is differentiable at a point if it has a derivative there. In other words, the function f is differentiable at x if

$$\lim_{h \rightarrow 0} (f(x+h) - f(x))/h$$

exists.

Find a limit

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{(x+h)^2 + (x+h) - (x^2 + x)}{h} = \\ & = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + x + h - x^2 - x}{h} = \lim_{h \rightarrow 0} \frac{2xh + h^2 + h}{h} = 2x + 1. \end{aligned}$$

$$f'(x) = 2x + 1.$$

The function $f(x) = x^2 + x$ is differentiable at $x = -1$:

$$f'(-1) = 2 \cdot (-1) + 1 = -2 + 1 = -1;$$

$$f'(-1) = -1.$$

Answer: Yes, the function $f(x) = x^2 + x$ is differentiable at $x = -1$.