Answer on Question # 84063, Math / Differential Geometry | Topology

Answer: $TS^n = S^n \times \mathbb{R}^n$ for n = 1, 3, 7.

A classical problem was to determine which of the spheres S^n are parallelizable. The zero-dimensional case S^0 is trivially parallelizable. The case S^1 is the circle, which is parallelizable as has already been explained. The hairy ball theorem shows that S^2 is not parallelizable. However S^3 is parallelizable, since it is the Lie group SU(2). The only other parallelizable sphere is S^7 . This was proved in 1958, by Michel Kervaire, and by Raoul Bott and John Milnor, in independent work. The parallelizable spheres correspond precisely to elements of unit norm in the normed division algebras of the real numbers, complex numbers, quaternions, and octonions, which allows one to construct a parallelism for each. Proving that other spheres are not parallelizable is more difficult, and requires algebraic topology.