

Answer on Question #83988 – Math – Geometry Question

The centroid of a triangle formed by the points $(0,0)$, $(\cos a, \sin a)$, and $(\sin a, -\cos a)$ lies on the line $y = 2x$, then a ?

Solution

The centroid is the intersection point of the medians. Line $y = 2x$ goes through the point $(0, 0)$ which is one of the vertices of the triangle. And also this line goes through the centroid by condition. This means that line $y = 2x$ is one of the medians of triangle.

Find the middle of the segment $(\cos a, \sin a)$ $(\sin a, -\cos a)$:

$$\left(\frac{\cos a + \sin a}{2}, \frac{\sin a - \cos a}{2} \right)$$

This point lies on the line $y = 2x$ which means:

$$\frac{\sin a - \cos a}{2} = 2 * \frac{\cos a + \sin a}{2}$$

$$\sin a - \cos a = 2 \cos a + 2 \sin a$$

$$3 \cos a + \sin a = 0$$

$$3 \cos a + \sqrt{1 - \cos^2 a} = 0$$

$$(1 - \cos^2 a) = 9 \cos^2 a$$

$$\cos^2 a = 1/10$$

$$\cos a = \pm 1/\sqrt{10}$$

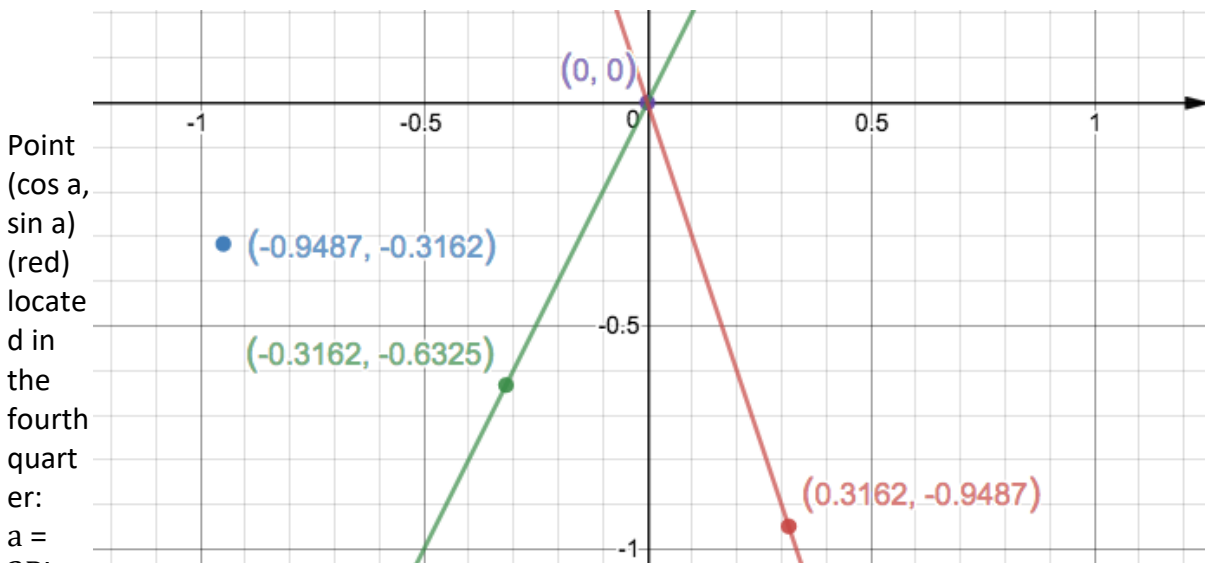
We got two solutions:

$$\cos a = 1/\sqrt{10}, \sin a = -3/\sqrt{10}$$

$$\cos a = -1/\sqrt{10}, \sin a = 3/\sqrt{10}$$

Draw solutions on the chart and find the angle a .

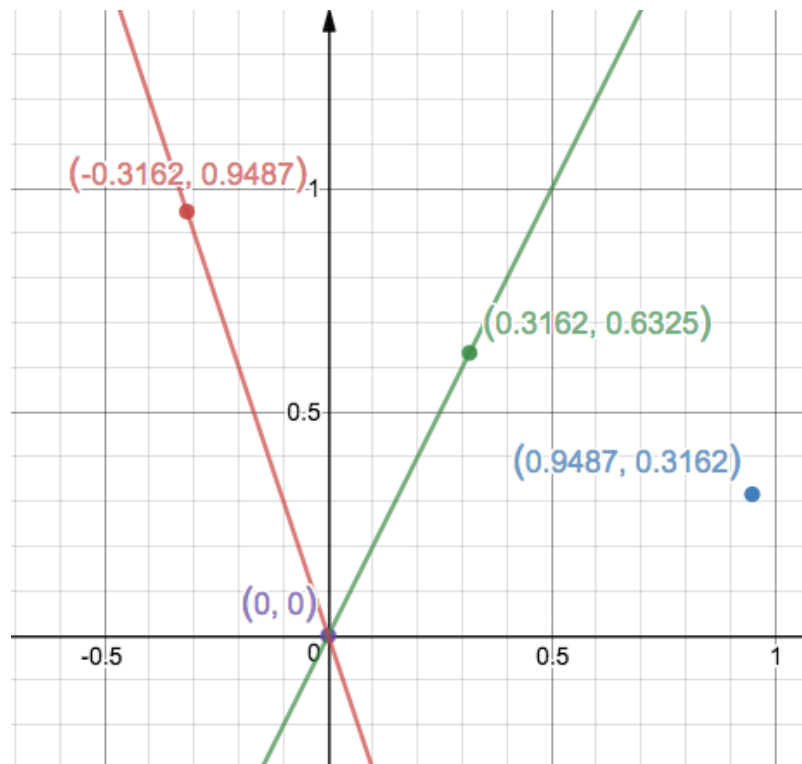
$$\cos a = 1/\sqrt{10}, \sin a = -3/\sqrt{10}$$



Point
 $(\cos a,$
 $\sin a)$
(red)
locate
d in
the
fourth
quart
er:

$$a = 2\pi - \cos^{-1}(1/\sqrt{10}) \approx 5.0341 \text{ rad}$$

$$\cos a = -1/\sqrt{10}, \sin a = 3/\sqrt{10}$$



Point $(\cos a, \sin a)$ (red) is in the second quarter:
 $a = \cos^{-1}(-1/\sqrt{10}) \approx 1.8925$ rad

Answer: $2\pi - \cos^{-1}(1/\sqrt{10}) \approx 5.0341$ rad, $\cos^{-1}(-1/\sqrt{10}) \approx 1.8925$ rad.