Answer on Question #83981 – Math – Analytic Geometry

Question

In a square $ABCD$, $A(1,3)$ and $C(4,2)$ are two vertices. AC is a diagonal. Express the coordinates of a point on the diagonal BD using a real parameter. Hence find the coordinates of the other two vertices.

Solution

The point $M\left(\frac{A_x+C_x}{2}, \frac{A_y+C_y}{2}\right) = M(2.5,2.5)$ is the center of the square.

The parametric equations of AC are $x = M_x + (C_x - A_x)t,\ y = M_y + (C_y - A_y)t.$

The parametric equations of BD are $x = M_x - (C_y - A_y)s,\ y = M_y + (C_x - A_x)s.$

There are relationships between vertexes:

$\overrightarrow{r_{BD}} = \overrightarrow{r_{AC}}^{\perp}$

$\overrightarrow{r_{OA}} = \overrightarrow{r_{OM}} + t_A\overrightarrow{r_{AC}}$

$\overrightarrow{r_{OC}} = \overrightarrow{r_{OM}} + t_C\overrightarrow{r_{AC}} = \overrightarrow{r_{OM}} - t_A\overrightarrow{r_{AC}},$

$\overrightarrow{r_{OB}} = \overrightarrow{r_{OM}} + s_B\overrightarrow{r_{BD}} = \overrightarrow{r_{OM}} + t_A\overrightarrow{r_{BD}},$

$\overrightarrow{r_{OD}} = \overrightarrow{r_{OM}} + s_D\overrightarrow{r_{BD}} = \overrightarrow{r_{OM}} - s_B\overrightarrow{r_{BD}}$

The coordinates of the vertex B are

$$x = M_x - (C_y - A_y)s_B = M_x - (C_y - A_y)t_A = M_x - \frac{(C_y - A_y)(A_x - M_x)}{(C_x - A_x)} = 2.5 - \frac{-1*(-1.5)}{3} = 2,$$

$$y = M_y + (C_x - A_x)s_B = M_x + (C_x - A_x)t_A = M_y + \frac{(C_x - A_x)(A_x - M_x)}{(C_x - A_x)} = 2.5 + \frac{\cancel{3}*(-1.5)}{\cancel{3}} = 1$$

The coordinates of the vertex D are

$$x = M_x - (C_y - A_y)s_D = M_x + (C_y - A_y)s_B = 2.5 + \frac{-1*(-1.5)}{3} = 3,$$

$$y = M_y + (C_x - A_x)s_D = M_y - (C_x - A_x)s_B = 2.5 - \frac{\cancel{3}*(-1.5)}{\cancel{3}} = 4$$

Answer:

The other two vertices are $B(2,1)$ and $D(3,4)$.