Answer to the question 83980, Math / Differential Equations

Denote by $\overline{v} = (v_1, v_2)$ the stream function. Since the flow is incompressible we have $div(\overline{v}) = 0$. Since it is potential we have that there exists the velocity potential u such that $v_1 = \frac{\partial u}{\partial x_1}$, $v_2 = \frac{\partial u}{\partial x_2}$. Thus from $div(\overline{v}) = 0$ we obtain that $\Delta u = 0$. As $v_1 = \frac{\partial u}{\partial x_1}$ we also have $\Delta v_1 = 0$. As $v_2 = \frac{\partial u}{\partial x_2}$ we also have $\Delta v_2 = 0$.

From $div(\overline{v}) = 0$ we have $\frac{\partial v_1}{\partial x_1} = -\frac{\partial v_2}{\partial x_2}$. From $v_1 = \frac{\partial u}{\partial x_1}$, $v_2 = \frac{\partial u}{\partial x_2}$ we have $\frac{\partial v_1}{\partial x_2} = \frac{\partial v_2}{\partial x_1}$. Thus we obtain also Cauchy-Riemann equations.