Answer on Question #83946 - Math - Trigonometry

Problem . If
$$f(x) = cos(x)$$
 and $x = 2\pi/15$. Prove that $16f(x)f(2x)f(4x)f(7x) = 1$
Proof. $f(x)f(2x)f(4x)f(7x) = cos(2\pi/15)cos(4\pi/15)cos(8\pi/15)cos(14\pi/15)$
Note that $cos(14\pi/15) = -cos(\pi/15)$
Therefore, we have
 $f(x)f(2x)f(4x)f(7x) = -cos(\pi/15)cos(2\pi/15)cos(4\pi/15)cos(8\pi/15)$
 $cos(\pi/15)cos(4\pi/15) = 1/2(cos(\pi/3) + cos(\pi/5)) = 1/2(1/2 + cos(\pi/5))$
 $cos(2\pi/15)cos(8\pi/15) = 1/2(cos(2\pi/3) + cos(2\pi/5)) = 1/2(-1/2 + cos(2\pi/5))$
So, $f(x)f(2x)f(4x)f(7x) = -1/4(1/2 + cos(\pi/5))(-1/2 + cos(2\pi/5)) =$
 $= -1/4(-1/4 - 1/2cos(\pi/5) + 1/2cos(2\pi/5) + cos(\pi/5)cos(2\pi/5)) =$
 $= -1/4(-1/4 - 1/2cos(\pi/5) + 1/2cos(2\pi/5) + cos(\pi/5)cos(2\pi/5)) =$
 $= -1/4(-1/4 - 1/2cos(\pi/5) + 1/2cos(3\pi/5))$
since $cos(2\pi/5) = cos(\pi - 3\pi/5) = -cos(3\pi/5)$
 $f(x)f(2x)f(4x)f(7x) = -1/4(-1/4) = 1/16$
So, $16f(x)f(2x)f(4x)f(7x) = 1$

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