

## Answer on Question #83946 - Math - Trigonometry

**Problem .** If  $f(x) = \cos(x)$  and  $x = 2\pi/15$ . Prove that  $16f(x)f(2x)f(4x)f(7x) = 1$

*Proof.*  $f(x)f(2x)f(4x)f(7x) = \cos(2\pi/15)\cos(4\pi/15)\cos(8\pi/15)\cos(14\pi/15)$

Note that  $\cos(14\pi/15) = -\cos(\pi/15)$

Therefore, we have

$$f(x)f(2x)f(4x)f(7x) = -\cos(\pi/15)\cos(2\pi/15)\cos(4\pi/15)\cos(8\pi/15)$$

$$\cos(\pi/15)\cos(4\pi/15) = 1/2(\cos(\pi/3) + \cos(\pi/5)) = 1/2(1/2 + \cos(\pi/5))$$

$$\cos(2\pi/15)\cos(8\pi/15) = 1/2(\cos(2\pi/3) + \cos(2\pi/5)) = 1/2(-1/2 + \cos(2\pi/5))$$

$$\text{So, } f(x)f(2x)f(4x)f(7x) = -1/4(1/2 + \cos(\pi/5))(-1/2 + \cos(2\pi/5)) =$$

$$= -1/4(-1/4 - 1/2\cos(\pi/5) + 1/2\cos(2\pi/5) + \cos(\pi/5)\cos(2\pi/5)) =$$

$$= -1/4(-1/4 - 1/2\cos(\pi/5) + 1/2\cos(2\pi/5) + 1/2\cos(\pi/5) + 1/2\cos(3\pi/5)) =$$

$$= -1/4(-1/4 + 1/2\cos(2\pi/5) + 1/2\cos(3\pi/5))$$

$$\text{since } \cos(2\pi/5) = \cos(\pi - 3\pi/5) = -\cos(3\pi/5)$$

$$f(x)f(2x)f(4x)f(7x) = -1/4(-1/4) = 1/16$$

$$\text{So, } 16f(x)f(2x)f(4x)f(7x) = 1$$

□