Answer on Question #83914 – Math – Calculus

Question

1. **1.** Sketch the curve C: y=xe^-x^2 and the line L: y=x+1 for 0 less than or equal to x less than or equal to 2. Hence, determine the area of the region bounded by the curve C, the line L, x=2 and the y-axis.

Solution

The figure 1 shows plots of the line L and the curve C. Obviously, the area desired

$$S = \int_0^2 (x+1)dx - \int_0^2 xe^{-x^2} dx$$

$$\int_0^2 (x+1)dx = \left(\frac{x^2}{2} + x\right)\Bigg|_0^2 = 4,$$

$$\int_0^2 x\,e^{-x^2} dx = \frac{1}{2}\int_0^2 e^{-x^2} dx^2 = \frac{1}{2}\left(-e^{-x^2}\right)\Big|_0^2 = \frac{1}{2}(1 - e^{-4})$$

$$S = 4 - \frac{1}{2}(1 - e^{-4}) = \frac{7}{2} + \frac{1}{2e^4}$$

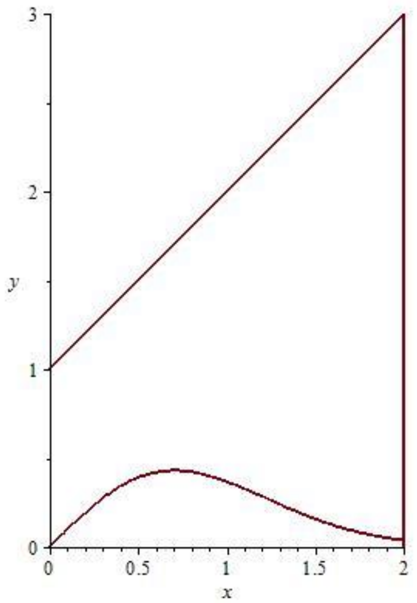

Figure 1.

Answer: $\frac{7}{2} + \frac{1}{2e^4}$.

Question

2. **2.** A cannon ball is shot up from the ground at an angle θ to the horizontal. The horizontal distance and the vertical distance from O against time, t, are governed by

the following equations respectively:
x = u cos(θ) t
y = u sin(θ) t – 0.5 g t^2
(Hint: u, θ and g are constant parameters)
Determine:

(i) dy/dx in terms of x, u, θ and g;
and
(ii) the maximum height of cannon ball attained.

Solution

(i)

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{u\sin\theta - gt}{u\cos\theta} = \frac{u\sin\theta - \frac{gx}{u\cos\theta}}{u\ \cos\theta} = \frac{u^2\sin\theta\cos\theta - gx}{u^2\cos^2\theta}$$

$$= \tan\theta - \frac{g}{u^2 cos^2\theta}x$$

(ii)

The maximum height is reached when

$$\frac{dy}{dt} = u\sin\theta - gt = 0 \to t = t_* = \frac{u\sin\theta}{g}$$

because $\frac{d^2y}{dt^2}(t_*) = (u\sin\theta - gt\,)'_t|_{t=t_*} = -g < 0$

Substitute $t = t_* = \frac{u\sin\theta}{g}$ into

$$y = u\,t\sin\theta - 0.5\ gt^2$$

We obtain

$$y_{max} = y(t_*) = \frac{u^2\sin^2\theta}{g} - \frac{u^2\sin^2\theta}{2g} = \frac{u^2\sin^2\theta}{2g}$$

Answer: i) $\frac{dy}{dx} = \tan\theta - \frac{g}{u^2 cos^2\theta}x$; **ii)** $y_{max} = \frac{u^2\sin^2\theta}{2g}$.